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Analysis of Price Equilibriums in the Aspen Economic Model under Various Purchasing Methods

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Abstract

Aspen, a powerful economic modeling tool that uses agent modeling and genetic algorithms, can accurately simulate the economy. In it, individuals are hired by firms to produce a good that households then purchase. The firms decide what price to charge for this good, and based on that price, the households determine which firm to purchase from. We will attempt to discover the Nash Equilibrium price found in this model under two different methods of determining how many orders each firm receives. To keep it simple, we will assume there are only two firms in our model, and that these firms compete for the sale of one identical good.

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Introduction and Definitions

Ever since ancient people first traded their produce with each other in a common square, humans have had an economy. As long as this economy has existed, there have been those who have tried to predict its behavior, and in order to do so, have created models of the economy as best they could. In our modern world, with stocks and exchanges and fiscal policies, it has become increasingly important to create realistic models of the economy in order to keep everything in balance and avoid worldwide disaster. Computers have given us the opportunity to create more complex models and to analyze them in much detail. However, with this powerful tool comes a greater need for care, and thus, we must be very precise in the creation and evaluation of our models. Any new tools that we can develop will prove invaluable over time. Fortunately, scientists have recently developed two new methods that allow us to greatly improve the accuracy of our models.

Agent Modeling

The first of these tools, agent modeling, has become increasingly popular over the years. Researchers have used it for everything from airplane flight simulation to the study of electricity outages. It has been of great use in economic modeling as well. The principle of agent modeling is that rather than programming the behavior of a large group, one should instead focus on the behavior of each individual, the way it makes its decisions, and how it interacts with others. The actions of these agents can then be combined to form more accurate large-scale behavior. In a model of an economy, agents may be individual households, or firms, or even the government. Several economic models based on agents have been created.

Genetic Algorithms

Genetic algorithms are founded on basic evolutionary principles. A population of genes is randomly defined that provides parameters for or a description of the system being modeled. These genes all begin with equal probability of occurring. At each time step or generation, one is chosen, and based on its success, the probability of its occurring again either increases or decreases. Genetic operators, such as crossovers and mutations, are used to create new genes, replacing those in the population that are not successful. Over time, the better possibilities eclipse the poorer ones, and the best courses slowly become set. Just as living creatures evolve the strongest behaviors over time, these algorithms determine the best method of achieving some goal, such as choosing the best price to charge consumers. In other words, the program can "learn" the most effective way of acting.

Aspen

Aspen is an economic model based on these two tools. Designed by scientists at Sandia National Laboratories, Aspen is unique in that rather than generating behavior of the economy as a whole, it models the behavior of individuals and single firms, and then aggregates the results to get macroeconomic behavior patterns. This allows for more detailed and realistic results, since agents much better represent real-world economy participants than some undifferentiated mob.

The original form of Aspen contains only three types of agents. These include households, firms, and a government. The government collects taxes and pays out unemployment. The head of each household works for some firm, producing a basic good that the households then buy. The firms produce only one good in this simple model, which represents some typical basket of goods needed by individuals. The amount of this good needed by each household depends on the size of each family. The firms learn through genetic algorithms how to price this good to maximize their profit, and the households decide through a probability method from which firm they should buy. In more complicated models, there are more types of goods and agents considered, and the pricing and buying methods are more complex.

Buying Methods

Assuming that a household experiences no search costs when it chooses which firm to buy from (in more current versions of Aspen, a search cost has been implemented), we can define the probability of buying from a particular firm based on the price it sets for its good in various ways. This report studies two different methods, the linear method and the inverse method. These will be explained later in the report.

Nash Equilibrium Price

If there is a price with the property that no firm can benefit by changing its price while the other firms keep their prices unchanged, then that price and the corresponding profit enjoyed by each firm constitute the Nash Equilibrium. In a realistic economy, the firms will naturally move towards the Nash Equilibrium without even having to communicate with one another, as they will learn over time that it is the best situation for them. Our goal in this report is to determine whether a Nash equilibrium price exists for each of the two buying methods, and if it does, then what the economy experiences at that price. Although in newer versions of Aspen, a range of prices may be charged by the firms in the equilibrium state, we will assume in this model that the equilibrium must have all firms charging the same price. We will find this price by trying to maximize the profit of each firm given that they must all charge the same price at this maximum.

Problem Definition

We are searching for a Nash equilibrium price in both the linear and inverse buying method models. Once we find these equilibriums (or determine that they do not exist), we will analyze the economy's behavior under two different purchasing methods during this equilibrium. We will then be able to determine the accuracy of each method in modeling realistic situations, based on whether the economy moves towards a Nash Equilibrium.

Theory

This section describes the two purchasing methods, finds the theoretical values for the Nash Equilibrium price under them, and analyzes these results. In order to have a simple model to analyze, we assume that there are only two firms that compete in the market for exactly one identical good, which all households need.

Inverse Method

This method determines the probability of purchasing from a given firm by inverting its price. Under this method, the probability P_1 that a consumer will purchase the good from Firm 1 at a given price p_1 is:

$$P_1 = kp_1^{-q}$$

The variable k is some normalizing constant used to ensure that the probabilities sum to 1. The variable q is an integer chosen by the user that determines how much of the business is allocated to the lower-priced firms. If the value of q is high, then there will be a low probability that a high-priced firm will have sell any of its goods. If the value of q is low, then this means that search costs are relatively high and even high-priced firms will receive some business. If households are willing to pay higher prices, the average price of the good will grow over time as the firms become aware of this fact. A graph displaying the final average price of the firms after 3,000 runs of the program for various values of q is in Figure 1. For this example, we set all the starting prices the firms charge to be $25.00.

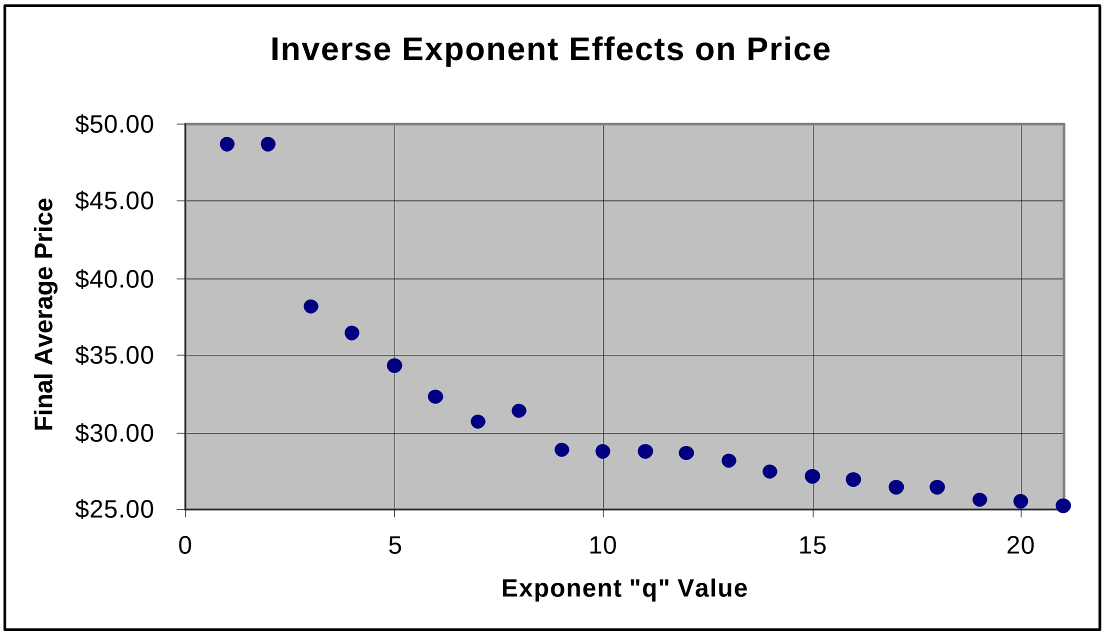

Figure 1. A Higher Exponent Gives a Lower Final Price

Nash Equilibrium Price Under Inverse Method

In order to determine the Nash equilibrium price, we will first define a function for the profit of each firm based on several variables, and then attempt to maximize this profit. We know that the total profit of a firm is the revenue minus the cost multiplied by the total number of products sold. Given that the wages and savings of the households are sufficiently high, we can safely assume that the demand for the product is constant. We can also assume that the firms will be able to supply all that the households demand (again, this can be made true based on the initial values of the variables). From that, we have the following equation for Firm 1's profit:

$$\Pi_1 = (p_1 - C) \cdot p_1^{-q} / (p_1^{-q} + p_2^{-q}) \cdot a$$

where

p_1 = price charged by first firm
p_2 = price charged by second firm
C = cost per product unit (wage / productivity)
q = demand exponent (as discussed in previous section)
a = entire market demand for product

The profit for Firm 2 can be found in the same way. To find the Nash Equilibrium price, that is, the price at which profit is maximized, we begin by taking the first derivative of Π_1 with respect to p_1.

$$\partial(\Pi_1) / \partial(p_1) = a\cdot(p_1^{-q})/(p_1^{-q} + p_2^{-q})$$
$$+ (p_1\text{-}C)\cdot(-qp_1^{-q-1})(p_1^{-q} + p_2^{-q})\text{-}(p_1^{-q})(-qp_1^{-q-1})/(p_1^{-q} + p_2^{-q})^2$$

Setting this expression equal to 0 and simplifying, we are left with:

$$p_1^{-2q} + (-q + 1)\, p_1^{-q} p_2^{-q} + Cqp_1^{-q-1}\, p_2^{-q} = 0$$

Now, since we assumed that there can be no price difference at the equilibrium, but that all the prices would be equal, we can set $p_1 = p_2$. Simplifying the result, we have:

$$(-q+2)\cdot p_1^{-2q} + Cqp_1^{-2q-1} = 0$$

Dividing by p_1^{-2q-1} and bringing one term to the other side, we have:

$$C\cdot q = (q\text{-}2)\, p_1$$

Solving for p_1, we have:

$$p_1 = C\cdot q / (q\text{-}2)$$

Thus, the Nash Equilibrium price is $p_1 = p_2 = C\cdot q/(q\text{-}2)$. So when both firms have set their price to this value, neither will benefit from raising or lowering its price. In a realistic economy, the firms should try to move to this point without even communicating among one another, as it will naturally benefit both of them.

Linear Method

The other method used determines the probability of purchasing from a given firm linearly. We define the probability P_1 that a consumer will purchase the good from Firm 1 at a given price p_1 to be:

$$P_1 = k(1 - (p_1)/(p_1+p_2)) = kp_2/(p_1+p_2)$$

where p_1 is the price charged by the first firm, and p_2 is the price charged by the second firm. Again, *k* is some normalizing constant used to ensure that the probabilities add up to 1.

Nash Equilibrium Price Under Linear Method

At this point, we can make two different assumptions that will affect the outcome.

Constant Demand

If we assume the market-wide demand is constant, which is valid given the savings and wages of consumers are high enough, then we have the following equation for the profit of Firm 1:

$$\Pi_1 = (p_1\text{-}C)\cdot(P_1)/(P_1 + P_2)\cdot a$$

We can quite similarly find an expression for the profit of Firm 2. Note that all variables are used as in the inverse method case. Simplifying the fraction with the probabilities, we see that $P_1 + P_2$ is equal to 1, and we have:

$$\Pi_1 = a \cdot (p_1\text{-}C) \cdot p_2/(p_1 + p_2)$$

Taking the first derivative of this equation with respect to p_1, simplifying it, and setting it equal to 0, we have:

$$\partial(\Pi_1) / \partial p_1 = a \cdot (p_2 p_1 + p_2^2 - p_1 p_2 + Cp_2)/(p_1+p_2)^2 = 0$$

Further simplifying:

$$p_2^2 + Cp_2 = 0$$

Recall that we have assumed all firms charge the same price at equilibrium. Thus, we may set $p_1 = p_2$, rewrite the above expression, and then solve for p_1 to get the equilibrium price:

$$p_1 = -C$$

Thus, we have found the Nash Equilibrium price to be $p_1 = p_2 = -C$. That is, all firms should charge -C dollars for their product to experience maximum benefit. This seems ludicrous, since no firm can charge a negative number (clearly, the cost per unit must be positive). This could mean that this method of purchasing is unrealistic, as no true Nash Equilibrium can ever be reached. We will explore this later in the report.

Variable Demand

Now, if we instead make the assumption that the market-wide demand depends on the wages and savings of the workers and the price of the goods, we can derive the following expression for profit. We will assume some average savings S for each worker and a constant wage W for each worker (set by the user). Therefore, the demand *a* can be expressed as:

$$a = 2(W+S)/(p_1+p_2)$$

which means the equation for profit under linear purchasing becomes:

$$\Pi_1 = (p_1\text{-}C) \cdot p_2/(p_1 + p_2) \cdot 2(W+S)/(p_1+p_2)$$

Taking the first derivative with respect to p_1 as before, setting it equal to 0, and simplifying, we get the equation:

$$p_2 (p_1+p_2)^2 - (p_1 p_2 - Cp_2) \cdot 2 \cdot (p_1 + p_2) = 0$$

Recalling that the firms must charge the same price at equilibrium, we set $p_1 = p_2$ and simplify to get:

$$4p_1^3 - 4p_1^3 + 4Cp_1^2 = 0$$

Solving for p_1:

$$p_1 = 0$$

Thus, the Nash Equilibrium price in this case is $p_1 = p_2 = 0$. This also seems puzzling. It appears as if there is no Nash Equilibrium price in this case either, since a firm cannot maximize its profit by charging $0.00 for each product. Apparently, no form of the linear purchasing method can be used to model a realistic economy. We will implement this method in the "Model Behavior" section to determine if a Nash Equilibrium is ever reached.

Model Behavior

We will study behavior in the model under both methods to determine the validity of the Nash Equilibrium found previously.

Inverse Method

We will now conduct a numerical experiment on the inverse method along with a full run of Aspen to determine whether the Nash Equilibrium price is exhibited in a real economy, and thus decide whether the inverse purchasing method could be used to model the economy.

Finding Profit At Nash Equilibrium

First, we will determine whether this price indeed maximizes profit, and whether both firms are benefited by it. Assuming a wage of $75.00 and a productivity of 3 units per worker per day, we find that $C = 25$. We also set q to 8. The starting price of the product is set to $25.00. By our formula, we should find that the profit maximizing price should be $C \cdot q / (q-2) = 25 \cdot 8/6 = \$33.33$. We now fix the price of Firm 1 at this amount, allowing the price Firm 2 charges to vary. At each price of Firm 2, we compute its profit per product unit using the equation given in the theory section (we just leave out the constant *a* used to represent the value of demand) and graph the results in Figure 2. We should see that Firm 2, and thus Firm 1, experiences greatest profit at $33.33.

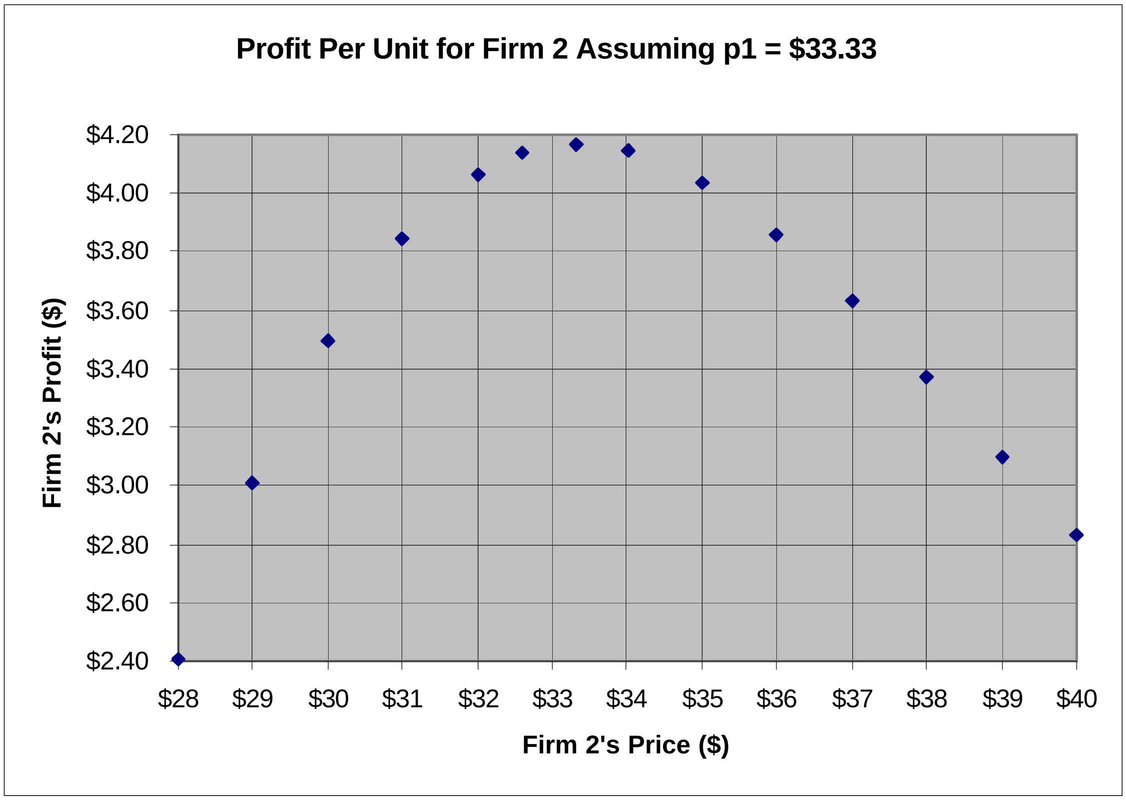

Figure 2. Nash Equilibrium Price in the Inverse Method

As we can see, Firm 2 experiences the greatest profit per unit (and thus, overall profit) if it matches its price to that of Firm 1, which is \$33.33, the equilibrium price. There is a peak at this point. It will experience no benefit if it sets its price higher or lower than this value.

Now, suppose p_2 is fixed slightly above or below this equilibrium value of \$33.33. Then Firm 2 will lower its price to meet that of Firm 1, attempting to create a balance, even though it cannot communicate with Firm 1. First, we set p_1 = \$30.00, slightly below the equilibrium price, and graph the profit of Firm 2 for various prices in Figure 3.

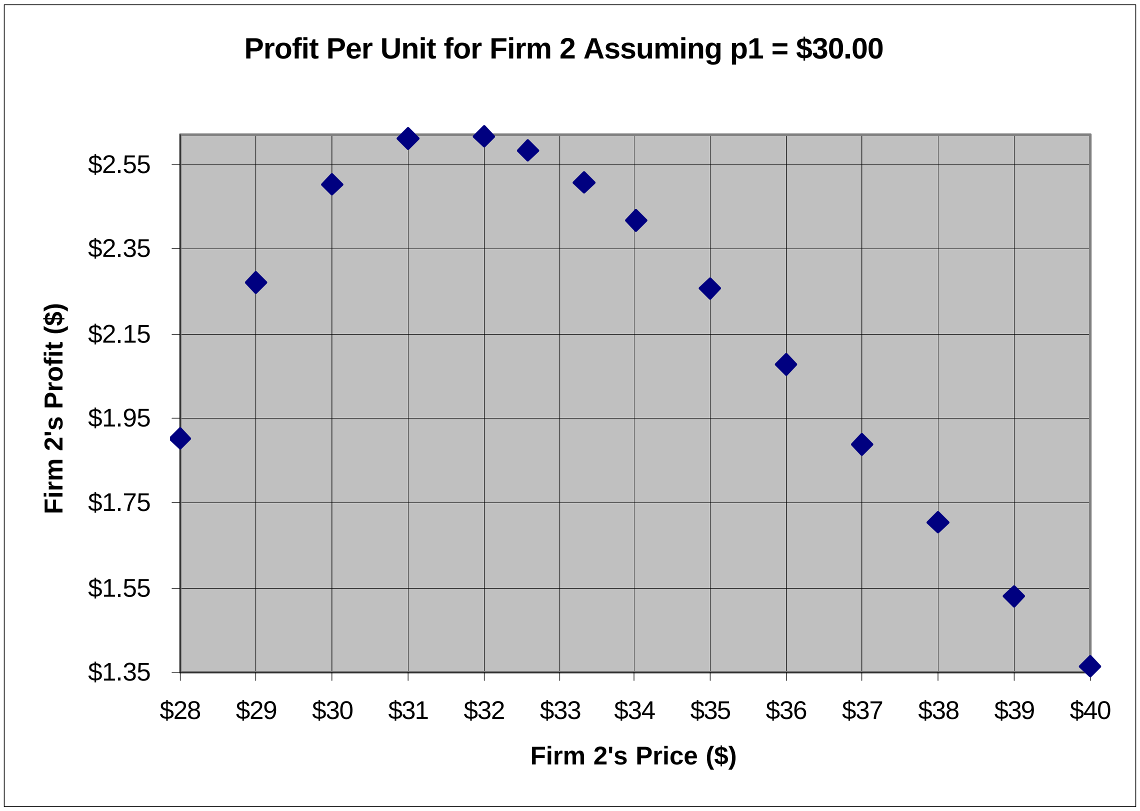

Figure 3. Adjusting for A Lower-Than-Equilibrium Price

First, note how the maximum profit per unit has greatly decreased. Before, when p_1 was set at equilibrium, it was around $4.50, but now it is only at $2.50. Also, the best price for Firm 2 to charge has moved down slightly, now being at $32.00. It achieves a best result at a lower price than in the previous case, since it wants to agree with Firm 1's price. If we were to raise p_1 to $35.00, slightly higher than then equilibrium price, and graph the profit of Firm 2 for various prices, we would see that the price Firm 2 charges would be slightly above $33.33, in order to agree with that of Firm 1.

Checking the Equilibrium in Aspen

Now, we will run the model to see whether the equilibrium price is achieved. We use the parameters defined in the Appendix, and with a quick calculation, see that the equilibrium price should be $33.33. Setting Aspen to run for 50,000 steps, we display the resulting average price at each step, and then take an average. The result is in Figure 4.

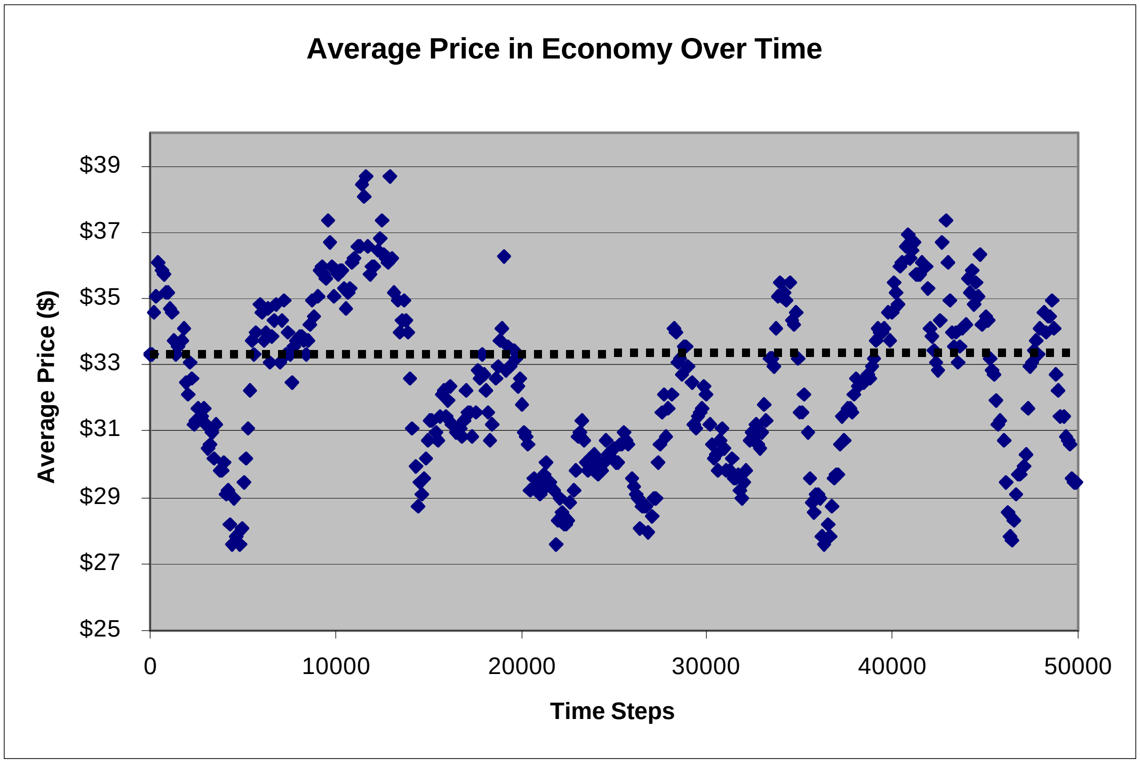

Figure 4. Nash Equilibrium Price in Aspen

The dashed line in the graph represents the average fitted line to the points, and is just above \$33.00, showing that our prediction was correct. In fact, ignoring all fluctuations in price due to business cycles, unemployment, and so on, we can see that the price fluctuates around this center equilibrium point. If we were to remove all influences, we would find that it in fact bottoms out to this value. Thus, we can conclude that our equilibrium price is valid, and that the model exhibits this price's benefits for the firms.

Linear Method

Now, we will repeat the above procedures for the two linear method cases. We must test whether the values found previously actually provide for a maximum benefit to both firms, and then we will use the Aspen model to see whether this equilibrium is ever actually reached.

<u>Finding Profit Under Nash Equilibrium</u>

We will test the validity of our strange results for the linear method. As in the inverse method case, we will find the profit for both constant and variable demand cases assuming a fixed price for Firm 1 of -\$25.00 for the constant method and \$0.00 for the variable method. For the constant method, when we plug in the values for p_1 and C, we have the equation:

$$(p_2-C)\,(-C/(p_2+C)) = -C.$$

Thus, the profit for Firm 2 when p_1 is set to the "equilibrium price" will always be -C dollars, regardless of the value of p_2. Clearly, that is not beneficial for Firm 2. Thus, this theoretical price equilibrium does not appear to be valid.

For the variable method, consider what happens when p_1 is set to the equilibrium price of \$0.00 computed earlier. We have the expression for the profit of Firm 2, with p_1 replaced with 0:

$$(p_2 - C) \cdot (0) / (0+p_2) \cdot 2 \cdot (W+S) / (0+p_2) = 0$$

Thus, the profit always equals \$0.00 for Firm 2 regardless of what price it charges, and regardless of how the demand varies. Again, it appears as if this is not a satisfactory Nash Equilibrium. We will analyze this later in the report.

Checking the Equilibrium in Aspen

We now wish to see whether an equilibrium price is found in Aspen under the linear method. We will not differentiate between the constant and variable method here, since the code for both is the same. We will make the savings and wages high enough that the households can initially buy all they need, and let the model run for 50,000 steps to see what results. A graph of the average price over time is in Figure 6.

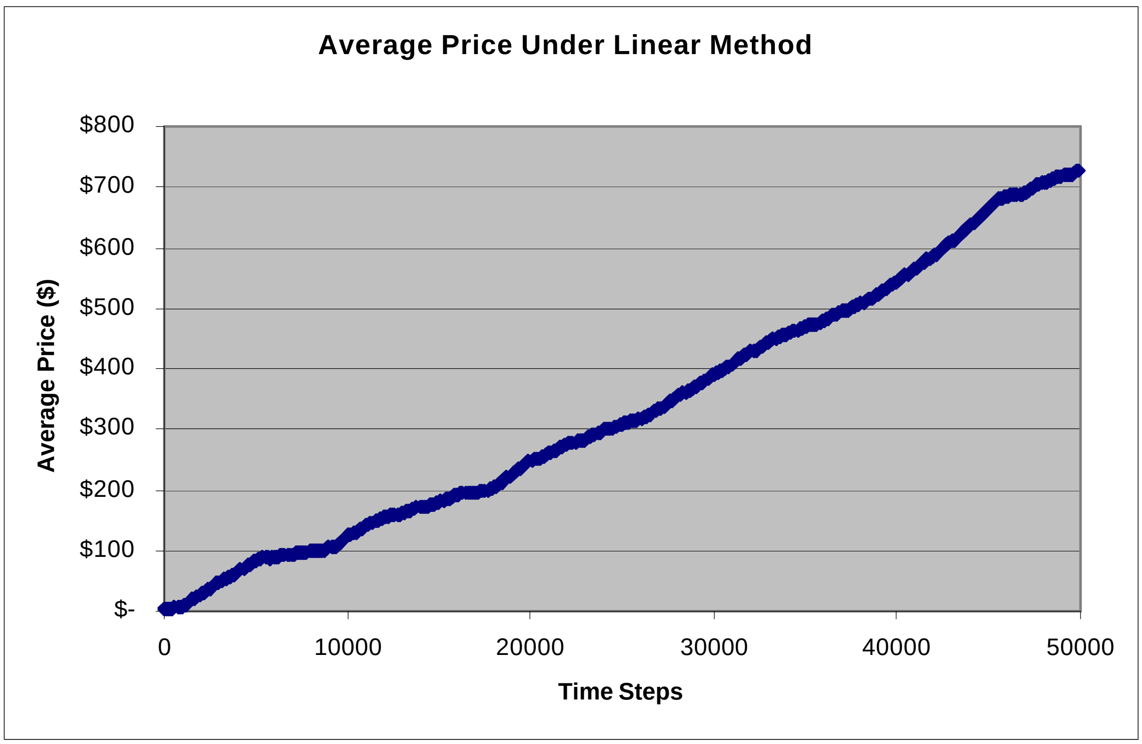

Figure 6. Search for a Nash Equilibrium Under Linear Method

The price rises steadily, with no fluctuation around a central equilibrium price. Under this method, no equilibrium price is ever found by the firms.

Analysis and Conclusions

Inverse Method

Through maximizing the profit equation, we found a Nash Equilibrium price with a value of Cq/(q-2) where C is the cost per product unit to the firm and q is a probability exponent defined by the user. We verified its validity by first setting p_1, the price charged by the first firm, to this value and allowing the price charged by the second firm to vary. In this case, we saw that this value did indeed fit the definition of a Nash Equilibrium price, since the second firm experienced greatest profit per product unit when it also charged the equilibrium price. When p_1 was set to other values, the optimal p_2 shifted accordingly to accommodate it.

When running the Aspen model, we found that although the results displayed some fluctuations that are quite natural in an economy, they oscillated around a median value that was actually equal to the value of the Nash Equilibrium price earlier computed and verified.

Thus, we can conclude that the theoretical value of the equilibrium price is correct in that it displays all the properties that a Nash Equilibrium price should. We can also conclude that the Aspen model can successfully find this equilibrium given enough time. Therefore, the inverse method could be used in an accurate simulation of economy.

Linear Method

The linear method is less encouraging. The theoretical values we found whether we considered demand to be constant or varying were nonsensical, since a firm can charge neither a negative price (equivalent to paying people to take the product) nor give away its products for free, and expect to experience any sort of profit. Also, when fixing p_1 at either equilibrium price and allowing p_2 to vary, we found that the profit of the second firm is either negative or zero regardless of the price, further lessening the validity of these values. Finally, when we ran Aspen under the linear purchasing method, we saw that the average price grew higher over time with no boundary, and did not fluctuate at all around any sort of central value. That is, no equilibrium was ever reached under this method.

Thus, we can conclude that the theoretical value of the equilibrium price is completely invalid and displays none of the appropriate properties. We can also conclude that this is not because of any fault of the method used to obtain it, but since the model failed to show any central price, it was simply the fact that under the linear purchasing method, there is no Nash Equilibrium price. Therefore, we can safely say that the linear purchasing method is totally unrealistic, and would not satisfactorily serve in an economic model such as Aspen.

Future Study

The Aspen model continues to improve. Although the inverse purchasing method appears to allow the model to run accurately, a new one is being developed in order to display even more realistic consumer behavior. In current research, search costs are being added into the model. Search costs simply come from the principle that consumers have limited time and resources, and thus will only search in a limited number of stores for the best price. In the model studied in this report, all price information is available to all households at all times, and thus a purchasing method such as the inverse method must be used to ensure that the higher-priced firms will have business. By including search costs, we allow the Nash Equilibrium price to evolve into a range of prices, thus better simulating the real world in which all firms do not benefit from being identical.

Appendix

For the two times that we ran Aspen in this report, we used the same initial values for the variables. The values follow. For a more complete explanation of the usage of these variables, see Sand Report SAND96-2459 [1].

Number of time-steps (days): 50,000

Households

Number of households: 1,000
Age of household head: 30 (Not used in this simpler version of Aspen)
Tax rate: 10%
Family size: Uniformly Distributed between 1 and 4 people
Savings: Uniformly Distributed between $1,000 and $5,000
Probability exponent (q): 8
Unemployed need fraction: 0.6
Residual need decay fraction: 0.5

Firms

Number of firms: 2
Worker wage rate: $75.00
Tax rate: 10%
Starting cash: Uniformly Distributed between $1,000 and $5,000
Worker productivity: 3 units per day
Initial product price:
  Inverse Method: $33.33
  Linear Method: $3.00
Initial inventory (units): 4,000
Minimum days inventory: 20 days worth
Employment increase factor: 1.10
Maximum days inventory: 40 days worth
Employment decrease factor: 0.90
Short lag constant: 5.0 (Used in Pricing)
Long lag constant: 10.0 (Used in Pricing)

Governments

Number of governments: 1
Unemployment benefit: $75.00
Starting cash: $1,000,000

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[1] Pryor, R.J., N. Basu, T. Quint, and T. Arnold, “Aspen: A Microsimulation Model of the Economy”, Sandia Report: SAND96-2459, Sandia National Laboratories, Albuquerque, NM, October 1996.

[2] Pryor, R.J., N.Basu, and T. Quint, “Development of Aspen: A Microanalytic Simulation Model of the US Economy”, Sandia Report: SAND96-0434, Sandia National Laboratories, NM, February 1996.

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