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Decentralized Control of Cooperative Robotic Vehicles

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ABSTRACT

This paper describes how decentralized control theory can be used to control multiple cooperative robotic vehicles. Models of cooperation are discussed and related to the input/output reachability and structural observability and controllability of the entire system. Whereas decentralized control research in the past has concentrated on using decentralized controllers to partition complex physically interconnected systems, this work uses decentralized methods to connect otherwise independent non-touching robotic vehicles so that they behave in a stable, coordinated fashion. A vector Liapunov method is used to prove stability of a single example: the controlled motion of multiple vehicles along a line. The results of this stability analysis have been implemented on two applications: a robotic perimeter surveillance system and self-healing minefield.

KEYWORDS: Mobile robotics, cooperative control, decentralized control.

1. INTRODUCTION

In recent years, there has been considerable interest in the control of multiple cooperative robotic vehicles. The vision being that multiple robotic vehicles can perform tasks faster and more efficiently than a single vehicle. This is best illustrated in a search and rescue mission where multiple robotic vehicles would spread out and search for a missing aircraft. During the search, the vehicles share information about their current location and the areas that they have already visited. If one vehicle's sensor detects a strong signal indicting the presence of the missing aircraft, it may tell the other vehicles to concentrate their efforts in a particular area.

Other types of cooperative tasks range from moving large objects [1] to troop hunting behaviors [2]. Conceptually, large groups of mobile vehicles outfitted with sensors should be able to automatically perform military tasks like formation following, localization of chemical sources, de-mining, target assignments, autonomous driving, perimeter control, surveillance, and search and rescue missions [3-6]. Simulation and experiments have shown that by sharing concurrent sensory information, the group can better estimate the shape of a chemical plume and therefore localize its source [7]. Similarly, for a search and rescue operation, a moving target is more easily found using an organized team [8-9].

In the field of distributed mobile robot systems, much research has been performed and summaries are given in [10][11]. The strategies of cooperation encompass theories from such diverse disciplines as artificial intelligence, game theory/economics, theoretical biology, distributed computing/control, animal ethology, and artificial life.

Most recently, researchers have begun to investigate using decentralized control techniques and graph theory to control multiple vehicles. Some simulations [12] have shown that a wireless network of mobile robots can be modeled as an undirected graph. In addition, Desai et al. [13-14] uses directed graph theory to control a team of robots navigating terrain with obstacles while maintaining a desired formation and changing formations when needed. Chen and Luh [15] examined decentralized control laws that drove a set of mobile robots into a circle formation. Similarly, Yamaguchi studied line-formations [16] and general formations [17], and so did Yoshida et al, [18]. Decentralized control laws using a potential field approach to guide vehicles away from obstacles can be found in [19-20]. Beni and Liang [21] prove the convergence of a linear swarm of distributed autonomous vehicles into a synchronously achievable configuration. The decentralized localization problem is examined by Roumeliotis and Bekey [22] and Bozorg et al. [23] via the use of distributed Kalman

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filters. Uchibe et al. [24] use Canonical Variate Analysis (CVA) for this same problem.

In this paper, we address the stable control of multiple vehicles using large-scale decentralized control techniques. The objective is to first analyze whether a large group of robotic vehicles, that is spread over a large spatial terrain, is input/output reachable and structurally controllable and observable. This depends on the communication paths available between vehicles and the information transmitted and received. Once we know that a system in structurally controllable and observable, we use provably asymptotically stable control laws to regulate the coordinated motion of the vehicles. The stability of these control laws is proven with a vector Liapunov technique.

The following section first describes the model of cooperation used in the analysis. This is followed by a stability analysis of the controlled motion of multiple vehicles along a straight. The remaining section discusses how this theory has been implemented on two test platforms.

2. MODEL OF COOPERATION

In this section, a group of robotic vehicles is modeled as a large dimensional interconnected system. It is a well-known fact that testing controllability and observability is a difficult numerical problem for large dimensions. Because of this, simple binary tests have been developed which test for input and output reachability and structural controllability and observability [25]. These tests are valid not only for the nominal nonlinear system but also for perturbed systems where the exact system parameters are unknown. Once controllability and observability have been assured, vector Liapunov techniques exist for testing asymptotic stability of the overall system. The analysis below shows some of the progress made in understanding how these techniques can be used in the design of large-scale distributed cooperative robotic vehicular systems.

Suppose that the overall system is denoted by

$$\mathbf{S}: \quad \begin{aligned} \dot{x} &= f(t,x,u) \\ y &= h(t,x) \end{aligned} \tag{1}$$

where $x(t) \in \Re^n$ is the state of $\mathbf{S}$ (e.g., x, y position, orientation, and linear and angular velocities of all vehicles) at time $t \in T$, $u(t) \in \Re^m$ are the inputs (e.g., the commanded wheel velocities of all vehicles), and $y(t) \in \Re^\ell$ are the outputs (e.g., GPS measured x,y position of all vehicles). The function $f: T \times \Re^n \times \Re^m \to \Re^n$ describes the dynamics of $\mathbf{S}$, and the function $h: T \times \Re^n \to \Re^\ell$ describes the observations of $\mathbf{S}$. We can partition the system into N interconnected subsystems given by

$$\mathbf{S}: \quad \begin{aligned} \dot{x}_i &= f_i(t,x_i,u_i) + \tilde{f}_i(t,x,u), \qquad i \in \{1,\dots,N\} \\ y_i &= h_i(t,x_i) + \tilde{h}_i(t,x) \end{aligned} \tag{2}$$

where $x_i(t) \in \Re^{n_i}$ is the state of the ith subsystem $\mathbf{S}_i$ at time $t \in \Re$, $u_i(t) \in \Re^{m_i}$ are the inputs to $\mathbf{S}_i$, and $y_i(t) \in \Re^{\ell_i}$ are the outputs of $\mathbf{S}_i$. The function $f_i: T \times \Re^{n_i} \times \Re^{m_i} \to \Re^{n_i}$ describes the dynamics of $\mathbf{S}_i$, and the function $\tilde{f}_i: T \times \Re^n \times \Re^m \to \Re^{n_i}$ represents the dynamic interaction of $\mathbf{S}_i$ with the rest of the system $\mathbf{S}$. The function $h_i: T \times \Re^{n_i} \to \Re^{\ell_i}$ represents observations at $\mathbf{S}_i$ derived only from local state variables of $\mathbf{S}_i$, and the function $\tilde{h}_i: T \times \Re^n \to \Re^{\ell_i}$ represents observation at $\mathbf{S}_i$ derived from the rest of $\mathbf{S}$. The *N* independent subsystems are denoted as

$$\mathbf{S}_i: \quad \begin{aligned} \dot{x}_i &= f_i(t,x_i,u_i), \qquad i \in \{1,\dots,N\} \\ y_i &= h_i(t,x_i). \end{aligned} \tag{3}$$

To determine input and output reachability and structural controllability and observability, we want to determine which inputs, outputs, and state variables affect each other through either a linear or non-linear relation. To perform this operation, it is convenient to write the state interconnection function as

$$\tilde{f}_i(t,x,u) = \tilde{f}_i\left(t, \bar{a}_{i1}x_1, \bar{a}_{i2}x_2, \dots, \bar{a}_{iN}x_N, \bar{b}_{i1}u_1, \bar{b}_{i2}u_2, \dots, \bar{b}_{iN}u_N\right), \quad i \in \{1,\dots,N\} \tag{4}$$

where the matrices $\bar{a}_{ij} \in B^{n_i \times n_j}$ and $\bar{b}_{ij} \in B^{n_i \times m_j}$ and the elements of the matrices are

$$\left(\bar{a}_{ij}\right)_{pq} = \begin{cases} 1, & \left(x_j\right)_q \ occurs\ in \left(\tilde{f}_i(t,x,u)\right)_p \\ 0, & \left(x_j\right)_q \ does\ not\ occur\ in \left(\tilde{f}_i(t,x,u)\right)_p \end{cases} \tag{5}$$

$$\left(\bar{b}_{ij}\right)_{pr} = \begin{cases} 1, & \left(u_j\right)_r \ occurs\ in \left(\tilde{f}_i(t,x,u)\right)_p \\ 0, & \left(u_j\right)_r \ does\ not\ occur\ in \left(\tilde{f}_i(t,x,u)\right)_p \end{cases} \tag{6}$$

where $q \in \{n_j\}$, $p \in \{n_i\}$, and $r \in \{m_j\}$. Similarly, the observation interconnection function may be written as

$$\tilde{h}_i(t,x) = \tilde{h}_i(t, \bar{c}_{i1}x_1, \bar{c}_{i2}x_2, ..., \bar{c}_{iN}x_N), \qquad i \in \{1,...,N\} \tag{7}$$

where $\bar{c}_{ij} \in B^{\ell_i \times n_j}$ and the elements of the matrix are

$$\left(\bar{c}_{ij}\right)_{zq} = \begin{cases} 1, & \left(x_j\right)_q \ occurs\ in \left(\tilde{h}_i(t,x)\right)_z \\ 0, & \left(x_j\right)_q \ does\ not\ occur\ in \left(\tilde{h}_i(t,x)\right)_z \end{cases} \tag{8}$$

where $q \in \{n_j\}$ and $z \in \{\ell_i\}$. Using these definitions, the interconnection matrix of $\mathbf{S}$ is a binary $(n+m+\ell)\times(n+m+\ell)$ matrix defined as

$$E = \begin{bmatrix} \bar{A} & \bar{B} & 0 \\ 0 & 0 & 0 \\ \bar{C} & 0 & 0 \end{bmatrix} \tag{9}$$

where the matrices $\bar{A} = \left(\bar{a}_{ij}\right)$, $\bar{B} = \left(\bar{b}_{ij}\right)$, and $\bar{C} = \left(\bar{c}_{ij}\right)$. The three rows and columns of the interconnection matrix represent the coupling between the state, input, and output variables. For large scale systems, the interconnection matrix E is often represented as a directed graph mapping state, input, and output variables from one subsystem to another subsystem. By searching this directed graph, it is possible to check for input and output reachability of the system [25]. Input reachability tells us if we can reach all the state variables from the input variables, while output reachability tells us if we can reach all the output variables from the state variables.

Mathematically it is possible to check for input and output reachability using the reachability matrix

$$R = E \vee E^2 \vee ... \vee E^s = \begin{bmatrix} F & G & 0 \\ 0 & 0 & 0 \\ H & \theta & 0 \end{bmatrix} \tag{10}$$

where $s = n + m + \ell$, $E^k = E^{k-1} \wedge E$, $\vee$ is the Boolean "or" operator $(0 \vee 0 = 0, 0 \vee 1 = 1 \vee 0 = 1 \vee 1 = 1)$, and $\wedge$ is the Boolean "and" operator $(1 \wedge 1 = 1, 0 \wedge 1 = 1 \wedge 0 = 0 \wedge 0 = 0)$. For two $s \times s$ binary matrices $A = (a_{ij})$ and $B = (b_{ij})$, the Boolean operations $C = (c_{ij}) = A \wedge B$ and $D = (d_{ij}) = A \vee B$ are defined by $c_{ij} = \bigvee_{k=1}^{s} (a_{ik} \wedge b_{kj})$ and $d_{ij} = a_{ij} \vee b_{ij}$.

The system $\mathbf{S}$ is input reachable if and only if the binary matrix G has no zero rows, and it is output reachable if and only if the binary matrix H has no zero rows. The system $\mathbf{S}$ is input-output reachable if and only if the binary matrix θ has neither zero rows nor zero columns. A system is structurally controllable if it is input reachable and the corresponding directed graph has no dilations, essentially meaning that there are enough input variables available to independently control all state variables. More formally, a directed graph $D = (U \cup X, E)$ is said to have a dilation if there exists a subset $X_k \subseteq X$, such that the number of distinct vertices of D from which a vertex in X_k is reachable, is less than the number of vertices of X_k. In this definition, the set of input variables is U, the set of state variables is X, and E is the set of edges connecting the set of vertices $U \cup X$. No dilation exist when the generic rank $\rho\left(\begin{bmatrix}\tilde{A} & \tilde{B}\end{bmatrix}\right) = n$ where $\tilde{A}$ and $\tilde{B}$ are the same as $\bar{A}$ and $\bar{B}$ except the "1" elements can take on any value. Similarly, a system is structurally observable if it is output reachable and the corresponding directed graph $D = (X \cup Y, E)$ has no dilations (i.e. generic rank $\rho\left(\begin{bmatrix}\tilde{A}^T & \tilde{C}^T\end{bmatrix}\right) = n$).

Feedback may be added to the system with

$$u_i = k_i(t, y_i) + \tilde{k}_i(t, y), \qquad i \in \{1,...,N\} \tag{11}$$

where the feedback interconnection function is given by

$$\tilde{k}_i(t, y) = \tilde{k}_i\left(t, \bar{k}_{i1} y_1, \bar{k}_{i2} y_2, ..., \bar{k}_{iN} y_N\right) \tag{12}$$

and $\bar{k}_{ij} \in B^{m_i \times \ell_j}$ and the elements of the matrix are

$$\left(\bar{k}_{ij}\right)_{rz} = \begin{cases} 1, & \left(y_j\right)_z \ \textit{occurs in} \left(\tilde{k}_i(t, y)\right)_r \\ 0, & \left(y_j\right)_z \ \textit{does not occur in} \left(\tilde{k}_i(t, y)\right) \end{cases} \tag{13}$$

where $r \in \{m_i\}$ and $z \in \{\ell_j\}$. With the feedback interconnection matrix denoted by $\bar{K} = \left(\bar{k}_{ij}\right)$, the system interconnection matrix becomes

$$E = \begin{bmatrix} \bar{A} & \bar{B} & 0 \\ 0 & 0 & \bar{K} \\ \bar{C} & 0 & 0 \end{bmatrix} \tag{14}$$

Again, the reachability matrix ($R = E \vee E^2 \vee ... \vee E^s$) may be used to determine input/output reachability and structural observability and controllability.

Note that in most prior research on decentralized control the state interconnection function $\tilde{f}_i(t, x, u)$ is non-zero, while the feedback interconnection function $\tilde{k}_i(t, y_i)$ is zero. In other words, typically it is desirable to stabilize a complex interconnected system using only decentralized controllers. However, in the case of multiple non-touching robotic vehicles, we have many non-interconnected systems, but we want to connect these systems through communication so that they behave in a coordinated fashion. For this case, the state interconnection function $\tilde{f}_i(t, x, u)$ is zero, and feedback interconnection function $\tilde{k}_i(t, y_i)$ is non-zero.

As an example, let us analyze a simple one-dimensional problem in which a linear chain of interdependent vehicles is to spread out along a line as shown in Figure 1. The objective is to spread out evenly along the line using only information from the nearest neighbor.

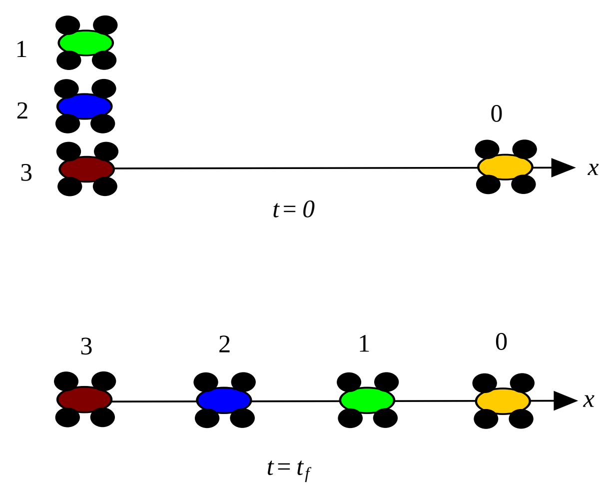


Figure 1. One-dimensional control problem. The top line is the initial state. The second line is the desired final state. Vehicles 0 and 3 are boundary conditions. Vehicles 1 and 2 spread out along the line by using only the position of their left and right neighbor.

Assume that the vehicle's plant is modeled as a simple integrator, and the commanded input is the desired velocity of the vehicle along the line. A feedback loop and a proportional gain K_p are used to control each vehicle's position. Figure 2(a) shows a block diagram of the control system. The dynamics of each subsystem is

$$\begin{aligned} \mathbf{S}_i: \quad & \dot{x}_i = -K_p x_i + K_p u_i, \qquad i \in \{1,\ldots,N\} \\ & y_i = x_i \end{aligned} \tag{15}$$

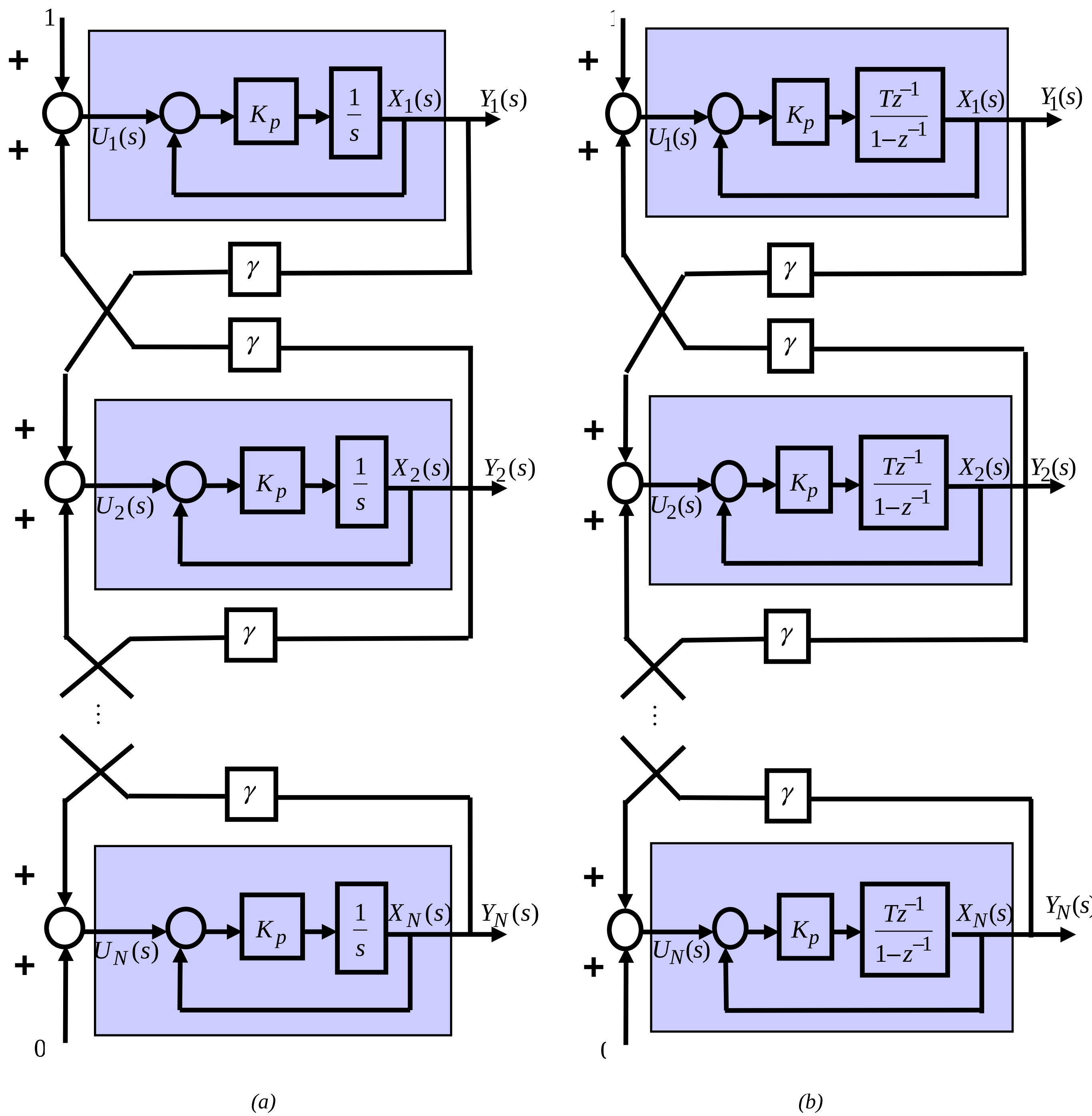


Figure 2. (a) Control block diagram of N-vehicle interaction problem. (b) Discrete time control block diagram of N-vehicle interaction problem.

where x_i is the position of the ith vehicle, u_i is the control input, and y_i is the observation. Assume the control of each vehicle is a function of the two nearest vehicles' observed positions, and the boundary conditions on the first and last vehicle are 1 and 0, respectively.

$$\begin{aligned} u_1 &= 1 + \gamma y_2 \\ u_i &= \gamma\left(y_{i-1} + y_{i+1}\right) \qquad i \in \{2, \ldots, N-1\} \\ u_N &= \gamma y_{N-1} \end{aligned} \tag{16}$$

where γ is the interaction gain between vehicles. The interconnection matrix of this system is

$$\overline{E} = \begin{bmatrix} \overline{A} & \overline{B} & 0 \\ 0 & 0 & \overline{K} \\ \overline{C} & 0 & 0 \end{bmatrix} \tag{17}$$

where $\overline{A} = \overline{B} = \overline{C} = I$,

$$\overline{K} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 1 & 0 & 1 & & 0 \\ 0 & 1 & 0 & & 0 \\ \vdots & & & \ddots & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \in B^{N \times N} \tag{18}$$

and I is the identity matrix of dimension $N \times N$. In this problem, the reachability matrix $R = E \vee E^2 \vee \ldots \vee E^s$ is a $3N \times 3N$ matrix of all ones, meaning that any state, input, or output can reach any other state, input, or output. Since the system is input and output reachable and there are no dilations, we know that the system is structurally observable and controllable.

3. STABILITY OF LARGE SCALE SYSTEMS

Once we know that a system is structurally observable and controllable, the next question to ask is that of connective stability. Will the overall system be globally asymptotically stable under structural perturbations? Analysis of connective stability is based upon the concept of vector Liapunov functions, which associates several scalar functions with a dynamic system in such a way that each function guarantees stability in different portions of the state space. The objective is to prove that there exist Liapunov functions for each of the individual subsystems and then prove that the vector sum of these Liapunov functions is a Liapunov function for the entire system.

To simplify matters, we will assume that the control function has already been chosen and the closed loop dynamics of the system can be written as

$$\mathbf{S}: \quad \dot{x}_i = g_i(t, x_i) + \tilde{g}_i(t, x), \qquad i \in \{1, \ldots, N\}. \tag{19}$$

The interconnection function can be written as

$$\tilde{g}_i(t, x) = \tilde{g}_i\left(t, \overline{e}_{i1} x_1, \overline{e}_{i2} x_2, \ldots, \overline{e}_{iN} x_N\right) \qquad i \in \{1, \ldots, N\} \tag{20}$$

where $\overline{e}_{ij} \in B^{n_i \times n_j}$, and the elements of the fundamental interconnection matrix $\overline{E} = \left(\overline{e}_{ij}\right)$ are

$$\left(\overline{e}_{ij}\right)_{pq} = \begin{cases} 1, & \left(x_j\right)_q \textit{ occurs in } \left(\tilde{g}_i(t, x, u)\right)_p \\ 0, & \left(x_j\right)_q \textit{ does not occur in } \left(\tilde{g}_i(t, x, u)\right)_p. \end{cases} \tag{21}$$

where $q \in \{n_j\}$ and $p \in \{n_i\}$.

The structural perturbations of **S** are introduced by assuming that the elements of the fundamental interconnection matrix that are one can be replaced by any number between zero and one, i.e.

$$e_{ij} = \begin{cases} [0,1], & \bar{e}_{ij} = 1 \\ 0, & \bar{e}_{ij} = 0. \end{cases} \tag{22}$$

Therefore, the elements e_{ij} represent the strength of coupling between the individual subsystems. A system is connectively stable if it is stable in the sense of Liapunov for all possible $E = (e_{ij})$ [25]. In other words, if a system is connectively stable, it is stable even if an interconnection becomes decoupled, i.e. $e_{ij} = 0$, or if interconnection parameters are perturbed, i.e. $0 < e_{ij} < 1$. This is potentially very powerful, as it proves that the system will be stable if an interconnection is lost through communication failure.

For linear systems such as in Figure 2(a), the linear system dynamics may be written as

$$\mathbf{S}: \quad \dot{x}_i = A_i x_i + \sum_{j=1}^{N} e_{ij} A_{ij} x_j, \qquad i \in \{1,\ldots,N\}, \tag{23}$$

and the Liapunov function for each individual subsystems is $v_i(x_i) = (x_i^T H_i x_i)^{1/2}$ where H_i is a positive definite matrix. For the system **S** to be connectively stable, the following test matrix $W = (w_{ij})$ must be an M-matrix (i.e., all leading principal minors must be positive) [25]:

$$w_{ij} = \begin{cases} \dfrac{\lambda_m(G_i)}{2\lambda_M(H_i)} - \bar{e}_{ii}\lambda_M^{1/2}\left(A_{ii}^T A_{ii}\right), & i = j \\ -\bar{e}_{ij}\lambda_M^{1/2}\left(A_{ij}^T A_{ij}\right), & i \neq j \end{cases} \tag{24}$$

where the symmetric positive definite matrix G_i satisfies the Liapunov matrix equation $A_i^T H_i + H_i A_i = -G_i$, and $\lambda_m(\bullet)$ and $\lambda_M(\bullet)$ are the minimum and maximum eigenvalues of the corresponding matrices.

In the example, the test matrix becomes

$$W = \begin{bmatrix} K_p & -K_p\gamma & 0 & \cdots & 0 \\ -K_p\gamma & K_p & -K_p\gamma & & \vdots \\ 0 & -K_p\gamma & K_p & & 0 \\ \vdots & & & \ddots & -K_p\gamma \\ 0 & \cdots & 0 & -K_p\gamma & K_p \end{bmatrix}. \tag{25}$$

For *N=2*, this test matrix is an M-matrix (i.e. the system is connectively stable) if $|\gamma| < 1$. For *N=3*, the system is connectively stable if $|\gamma| < \frac{1}{\sqrt{2}}$. For *N=4*, the system is connectively stable if $|\gamma| < 0.618$. Notice how the range of the interaction gain gets smaller for larger sized systems. In fact, for this particular example, the interaction gain range reaches a limit of $|\gamma| \leq 0.5$ for infinite numbers of vehicles.

This same analysis can also be performed in the discrete domain [26]. Consider a discrete dynamic system described by

$$\mathbf{S}: \quad x_i(k+1) = A_{ii} x_i(k) + \sum_{j=1}^{N} e_{ij} A_{ij} x_j(k), \quad i \in \{1,\ldots,N\} \tag{26}$$

and a Liapunov function $v_i(x_i) = (x_i^T H_i x_i)^{1/2}$. The test matrix is

$$w_{ij} = \begin{cases} \xi_i, & i = j \\ -e_{ij}\xi_{ij}, & i \neq j \end{cases} \tag{27}$$

where $\xi_i = 1 - \sqrt{1 - \dfrac{1}{\lambda_M(H_i^*)}}$, $\xi_{ij} = \lambda_M^{1/2}\left(A_{ij}^T A_{ij}\right)$, and $A_{ii}^T H_i^* A_{ii} - H_i^* = -I$, and the superscript * denotes the Hermitian operator.

Inserting a zero order hold function before the integrator in Figure 2(a), we can transform our example problem above into the discrete time domain as shown in Figure 2(b). The sampling period is denoted by *T*. The sampling period is both the communication and position update sample time. The state equations of the system are

$$\mathbf{S}: \quad \begin{aligned} x_1(k+1) &= (1-K_pT)x_1(k) + \gamma K_pTx_2(k) \\ x_i(k+1) &= (1-K_pT)x_i(k) + \gamma K_pTx_{i-1}(k) + \gamma K_pTx_{i+1}(k), \quad i \in \{2,\ldots,N-1\} \\ x_N(k+1) &= (1-K_pT)x_N(k) + \gamma K_pTx_{N-1}(k) \end{aligned} \tag{28}$$

If $0 < K_pT \le 1$, the resulting test matrix is

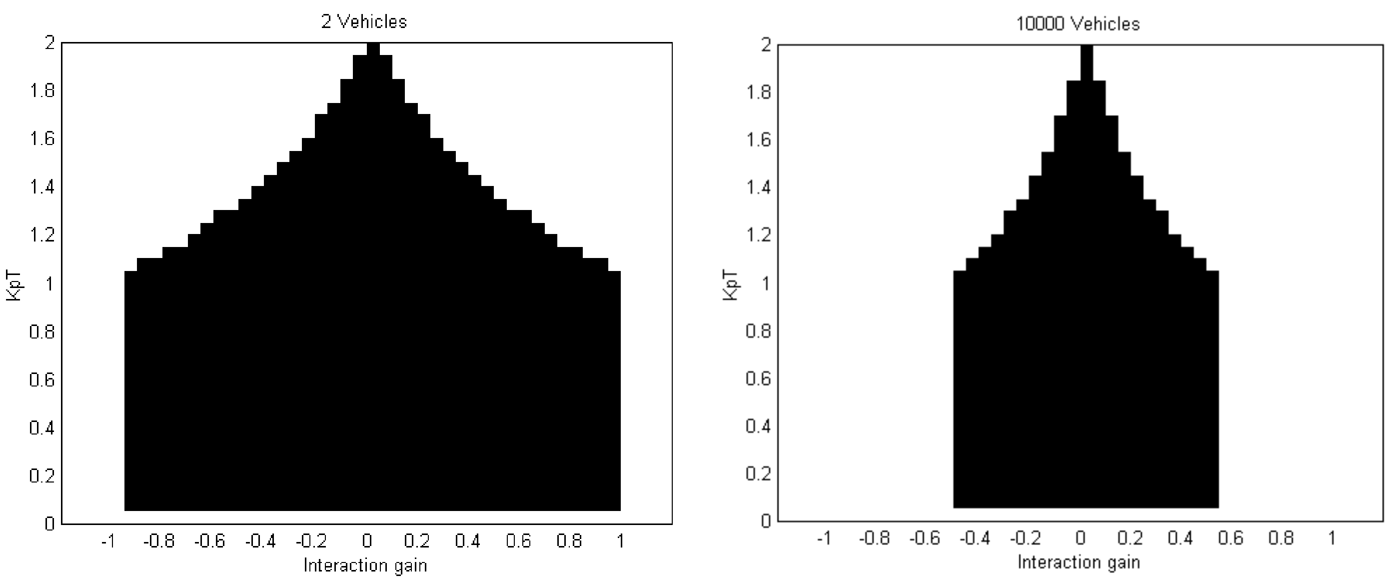
$$W = \begin{bmatrix} K_pT & -K_pT\gamma & 0 & \cdots & 0 \\ -K_pT\gamma & K_pT & -K_pT\gamma & & \vdots \\ 0 & -K_pT\gamma & K_pT & & 0 \\ \vdots & & & \ddots & -K_pT\gamma \\ 0 & \cdots & 0 & -K_pT\gamma & K_pT \end{bmatrix}, \tag{29}$$

and if $1 < K_pT \le 2$, the test matrix is

$$W = \begin{bmatrix} (2-K_pT) & -K_pT\gamma & 0 & \cdots & 0 \\ -K_pT\gamma & (2-K_pT) & -K_pT\gamma & & \vdots \\ 0 & -K_pT\gamma & (2-K_pT) & & 0 \\ \vdots & & & \ddots & -K_pT\gamma \\ 0 & \cdots & 0 & -K_pT\gamma & (2-K_pT) \end{bmatrix} \tag{30}$$

For *N=2*, the test matrix is an M-matrix, and the system is connectively stable if

$$|\gamma| < \begin{cases} 1, & 0 < K_pT \le 1 \\ \dfrac{2}{KT} - 1, & 1 < K_pT \le 2 \end{cases} \tag{31}$$



(a) *(b)*

Figure 3. Stability region for the (a) N=2 vehicles and (b) N=10000 vehicles.

Figure 3(a) illustrates the stability region for the case of *N=2*. The dark region represents stable combinations of the interaction gain γ and K_pT (proportional control gain multiplied by the sampling period). The white region represents unstable combinations of γ and K_pT. We refer to the dark region as a stability "house" due to the shape of the stable zone. The size of this stability house varies only with *N*. As *N* is increased, the house gets smaller in width but maintains the same height and shape. Figure 3(b) shows the stability region for *N=10000*.

For this particular example, another way to check the stability of this linear system is to check that the eigenvalues of the system matrix *A* are within the unit circle. There is a special formula (p. 59 of [27]) for the eigenvalues of *A* given by

$$\lambda_i(A) = 1 - K_pT + 2K_pT\gamma\cos(\frac{i\pi}{N+1}), \quad i = 1, \cdots, N \; . \tag{32}$$

From this formula, we can see that as $N \to \infty$ the cosine term becomes unity. This implies that γ must stay between –0.5 and 0.5 for K_pT less than one in order to maintain stability. For K_pT greater than one, the admissible γ values taper off parabolically (the sloped "roof") until $K_pT = 2$.

It must be remembered that the above example assumed that the sampling period for both communication and position are the same. It can be shown that if the position sampling period is much less than the communication sampling period *T*, then the stability region is independent of T and only dependent on the interaction gain γ. In the limit, the position feedback loop may be modeled as a continuous time system, and the zero order hold may be moved outside the position feedback loop. As long as the position feedback loop is stable ($K_p > 0$), then there will be no overshoot in driving the vehicle, and the vehicle will stop at the desired position given by $\gamma(x_{i-1} + x_{i+1})$ at each communication sample period. Intuitively this result is obvious.

Several conclusions can be drawn from this stability analysis. First, asymptotic stability of vehicle positions depends on vehicle responsiveness K_p, communication sampling period *T*, and vehicle interaction gain γ. If the vehicle is too fast (large K_p) or the sample period is too long (large *T*) then the vehicles will go unstable. There is a dependence on interaction gain for stability as well. Second, the interaction gains can be used to bunch the vehicles closer together or spread them out. Third, the stability region shrinks as the number of vehicles, *N*, increases but only to a defined limit.

4. EXPERIMENTAL TEST PLATFORMS

The stability analysis described in the previous section has been implemented on two robotic vehicle platforms. The first platform uses 4 RATLER™ vehicles to guard a perimeter as described in [28-31]. In this application, the line that the vehicles are to be controlled on is the curved perimeter shown in Figure 4. The RATLER vehicles, shown in Figure 5, guard the perimeter by attending to alarms from intrusion detection sensors. When not attending to alarms, the vehicles position themselves along the perimeter at one-half the distance between the two nearest neighbors on each side. This corresponds to an interaction gain of 0.5 in the analysis in the previous section. Differential GPS is used to locate and guide each vehicle. A RF radio on each vehicle is used to broadcast its GPS position to the others. Each vehicle has a communication time slot of 220 milliseconds, which results in a total communication sample period of 1.10 seconds for 4 vehicles and a base-station. The differential GPS sample period is 200 milliseconds. As the previous section points out, stable control is guaranteed as long as the differential GPS sample period is faster than the communication sample period, and the vehicle has a faster inner position control loop based on the GPS position.

The second application where this analysis has been used is in the position of mobile landmines in a self-healing minefield. In this application, the mobile landmines are to fill in the lane left when the field is breached as shown in Figure 6. The mobile landmines use the same algorithm used by the perimeter robots. When a mine is detected missing, the remaining mines position themselves so that they are one-half the distance between the neighboring mines. Again, the interaction gain is 0.5. This algorithm has been tested on ten robot vehicles similar to the one shown in Figure 7. When one vehicle is turned off, the others notice the missing vehicle and adjust their position to fill in the breach. These vehicles use an omni-direction ultrasound sensor and a RF radio to determine their position amongst each other. Each vehicle broadcasts its position every 100 milliseconds, which results in a total communication and position sample period of 1 second. In this case, an inner position control loop does not exist, so the system will only stabilize if the responsiveness of each vehicle within one second does not overshoot its desired position.

Figure 4. Perimeter being guarded by robot sentries.

Figure 5. RATLER vehicles around the laptop base-station.

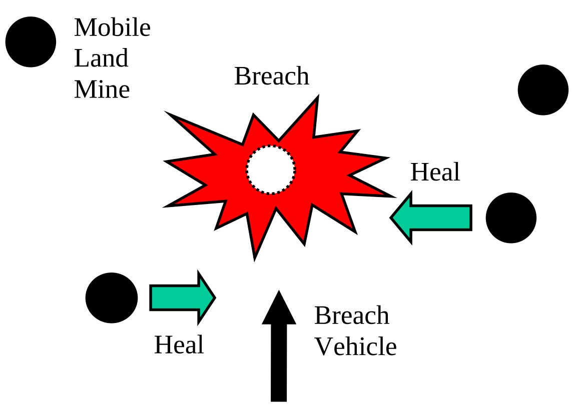


Figure 6. Self-healing minefield.

Figure 7. Robotic vehicles used to test self-healing minefield.

5. CONCLUSIONS

In this paper, decentralized control theory is applied to the control of multiple cooperative mobile robotic vehicles. We mathematically described how to determine if a cooperative system is input/output reachable, structurally controllable and observable, and connectively stable. We illustrated the use of these techniques on a simple problem, and we showed how this simple example is applicable to both perimeter surveillance and self-healing minefield problems. The stability analysis was used to determine limits on system parameters such as the interaction gain between vehicles, on the responsiveness of the vehicles, and on the sampling period for communication and position feedback, and to see how these limits vary as a function of the number of vehicles.

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