Modeling Multi-Material Systems with the Extended Finite Element Method

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Explicit Eulerian and Arbitrary Lagrangian Eulerian (ALE) codes are often used to model large deformation mechanics where large vortices make a Lagrangian approach problematic. Whereas Lagrangian Finite Element codes can maintain distinct material interfaces at element boundaries, this is not true for ALE simulations. With respect to the discussions that follow, an ALE algorithm advances a solution in a two-step (operator-split) process. First, the standard Lagrangian-step is taken. At this point, the data are moved (remapped) in a mass and energy conserving way to an "improved" mesh. Material interfaces that may have corresponded to element boundaries now lie in their interiors. These intra-element material boundaries are frequently treated via *ad hoc* models [1]. Various models have been used but generally fail as they (i) impose a single velocity field on the mixed element and (ii) model interface physics in a volumetric rather than surface-phenomenological way. In the first case, bad material states may be reached as the materials share a common deformation. In the second case, spurious material bonding may occur. Here we describe advancement in operator-split ALE technology that uses the eXtended Finite Element Method (X-FEM) to i) provide multiple kinematic fields per multi-material element and ii) define surfaces across which the materials can interact. These algorithms provide increased physics fidelity for a range of Sandia's ALE missions such as the evaluation of armor/anti-armor effectiveness and weapons performance.

The X-FEM provides a methodology to enhance standard Finite Element bases while retaining their underlying convergence properties [2]. Here we allow for discontinuities in the displacement fields via Heaviside enrichment functions aligned with material boundaries. In this way, each material in a multi-material element has its own kinematic field. Additionally, a discontinuity surface results through which the materials interact (e.g., via a Coulomb friction model). The Lagrangian portion of the ALE X-FEM algorithm draws from the explicit dynamics X-FEM literature (e.g., [2]). Specifically, it has been shown that a multi-material element may be decomposed into multiple single-material elements, one per material. This decomposition is demonstrated in Figure 1 for a two-material element. In the illustration, the multi-material X-FEM element is equivalent to two partially-filled single-material elements. These single-material elements are found in operator-split ALE codes. With the proper choice of mass matrix, this formulation has a number of beneficial features, including (i) a stable time-step that depends on the parent element length scale and (ii) use of much of the existing Lagrangian-step framework of the host ALE code. This formulation is inspired by Reference 1 but also differs in several ways. Here, we enforce interface constraints on intra-element discontinuities via Lagrange multipliers. In particular, we provide for intra-element interface constraints using the Forward Increment Lagrange Multiplier approach [3]. A beneficial feature of this technique is that the stable time-step is not modified relative to the unconstrained case. These interfaces are constructed via the second-order accurate Patterned Interface Reconstruction (PIR) algorithm that greatly reduces element-to-element interface segment discontinuities, relative to other schemes. The final ingredient is an unsplit remap that serves to move field data from the end of one X-FEM Lagrange step to the start of a new step.

An example of the method's efficacy is illustrated in Figure 2. Here a leftward-moving cylinder impacts a pair of plates. Materials are inserted into an empty domain (blue) and the X-FEM algorithm generates single-material elements and intra-element surfaces. As the solution advances, searches for intra-element interface interactions are made, and frictionless constraints are enforced. The center figure demonstrates that cylinder and plate rebound after impact, while the right-hand image illustrates spurious bonding typical of a non-X-FEM simulation.

References

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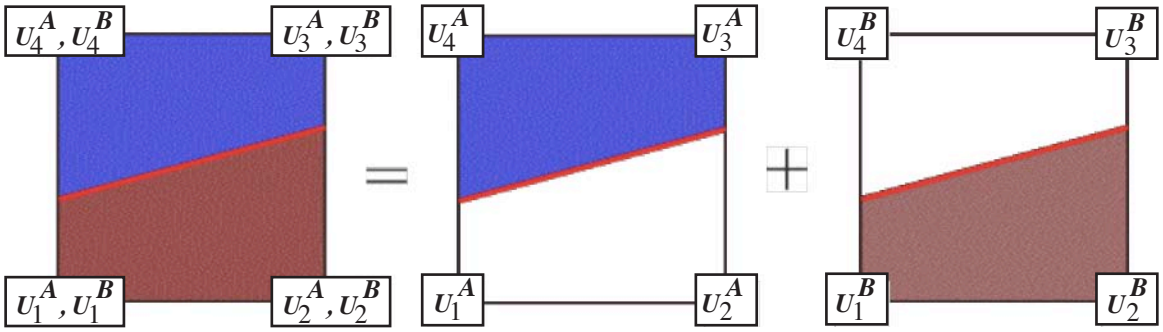

Figure 1. A Heaviside eXtended Finite Element enrichment of a multiple material element is identical to the composition of two single material elements. These elements are available in most operator-split ALE codes.

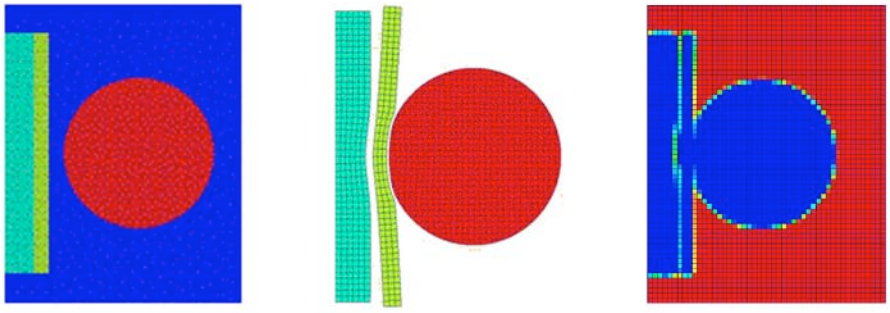

Figure 2. Right-hand-side shows two X-FEM plates and cylinder inserted in an empty domain. The cylinder is given a left-ward velocity and the center image shows its rebound. The right-most image demonstrates cylinder-plate bonding for a non-X-FEM simulation.