SAND99-2859C

The Use of Reproducing Kernel Functions for Discrete Filtering in Large-Eddy Simulation†

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Fifth U.S. National Congress on Computational Mechanics
Boulder, Colorado

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† Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under Contract DE-AC04-94AL85000.

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Overview

- Background and Motivation
- Research Objectives
- Reproducing Kernel Particle Method
  - Formulation
  - Kernels
- Commutivity Error of Filters
- Spectral Behavior of Filters
- A Priori DNS Realizability Tests
- Conclusions

This Work is Quantifying the Performance of RKPM Filters for LES

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The objective of this work is to study the use of RKPM kernels for Large Eddy Simulation on unstructured FEM grids.

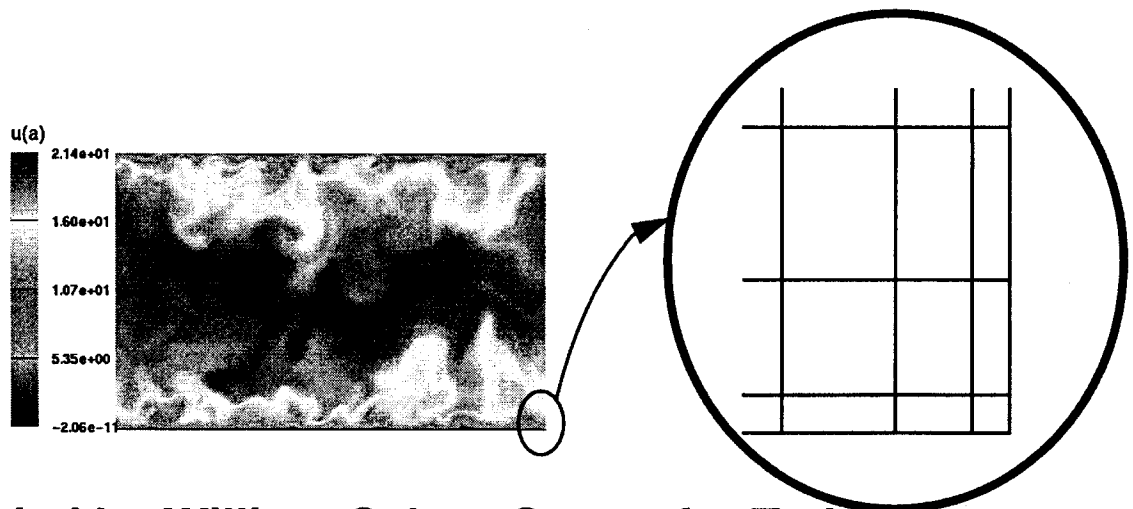
Filter Requirements:

- Control commutivity error
- Meet realizability conditions
- Treat finite domains with unstructured grids.

DNS Data*

$Re_\tau = 395$

192x192x256



* DNS data provided by William Cabot, Center for Turbulence Research Stanford University.

To Minimize Errors Associated with LES, Filters Must Meet Several Requirements

- **Commutivity Errors Must Be Minimized:**

$$\left[\frac{d\psi}{dx}\right] \equiv \overline{\frac{d\psi}{dx}} - \frac{d}{dx}\overline{\psi} \qquad \overline{\psi} = \sum_{i=1}^{Nsup} G_i \psi_i$$

- **Realizability Conditions for the Turbulent Stress Tensor Must Be Satisfied:**

$$I_1 \equiv \tau_{11} + \tau_{22} + \tau_{33} > 0$$

$$I_2 \equiv \tau_{11}\tau_{22} - \tau_{12}^2 + \tau_{11}\tau_{33} - \tau_{13}^2 + \tau_{22}\tau_{33} - \tau_{23}^2 > 0$$

$$I_3 \equiv det([\tau]) \geq 0$$

- **Filter Attenuation Properties Must Be Understood.**

The Reproducing Kernel Method Provides a Natural Framework for LES Filtering

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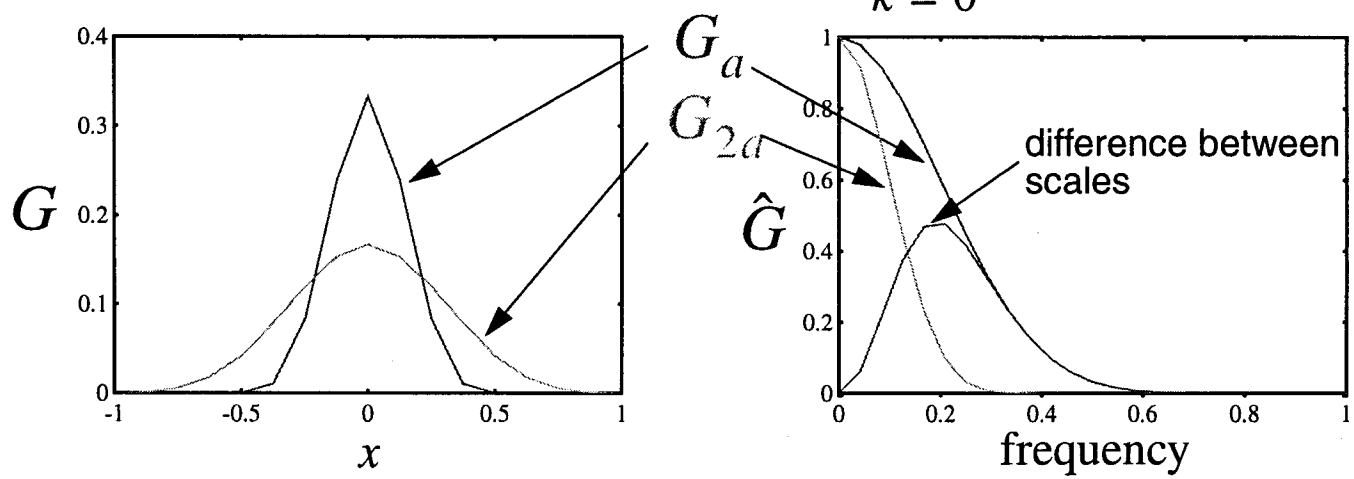
- **Compact Support**
- **Partition of Unity**
- **Variable Δx and boundaries**
- **Commutivity Error Control**
- **Scale Separation**

$$\bar{f}(x_j) = \sum_{i=1}^{Nsup} G_i^a(x_j) f_i$$

$$1 = \sum_{i=1}^{Nsup} G_i^a(x_j)$$

$$G^a(x - x_i) = \sum_{k=0}^{S} \beta_k \varphi_i^a(x)(x - x_i)^k$$



Commutivity Errors for RKPM-Based Filters May Be Expressed Analytically

Commutivity error is defined as:

$$\left[\frac{\partial f}{\partial x}\right] \equiv \overline{\frac{\partial f}{\partial x}} - \frac{\partial}{\partial x}\bar{f}$$

Kernel function moments are given as:

$$m_k(x) \equiv \sum_{i=1}^{Nsup} \left(\frac{x - x_i}{\Delta x}\right)^k G_i^a(x)$$

Spatial derivative of filtered function is:

$$\frac{\partial}{\partial x}\bar{f} = \sum_{k=0}^{\infty} \frac{(-\Delta x)^k}{k!} m_k(x) f^{(k+1)}(x) + \sum_{k=0}^{\infty} \frac{(-\Delta x)^k}{k!} m_k^{(1)}(x) f^{(k)}(x)$$

Filtered spatial derivative is:

$$\overline{\frac{\partial f}{\partial x}} = \sum_{k=0}^{\infty} \frac{(-\Delta x)^k}{k!} m_k(x) f^{(k+1)}(x)$$

Commutivity Errors for RKPM-Based Filters May Be Expressed Analytically

Yields Commutivity Error in terms of moment derivatives:

$$\left[\frac{\partial f}{\partial x}\right] = \sum_{k=0}^{\infty} \frac{(-\Delta x)^k}{k!} m_k^{(1)}(x) f^{(k)}(x)$$

Consistency to p yields:

$$m_0^{(1)} = \ldots = m_p^{(1)} = 0,\ m_{p+1}^{(1)} \neq 0$$

Finally, because:

$$m_{p+1}^{(1)} = O\left(\frac{1}{\Delta x}\right)$$

Commutivity Error Converges as:

$$\left[\frac{\partial f}{\partial x}\right] = O(\Delta x^p)$$

Our Filters Take Advantage of Nodal Connectivity on an Unstructured Grid

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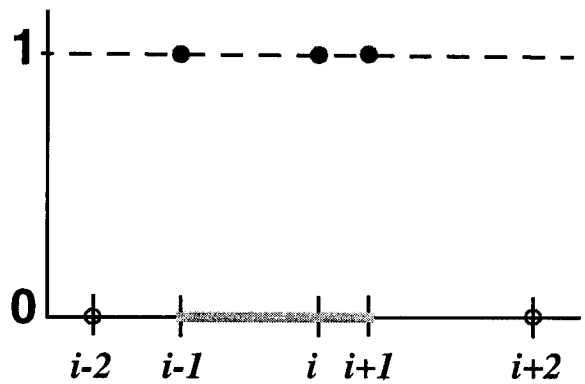
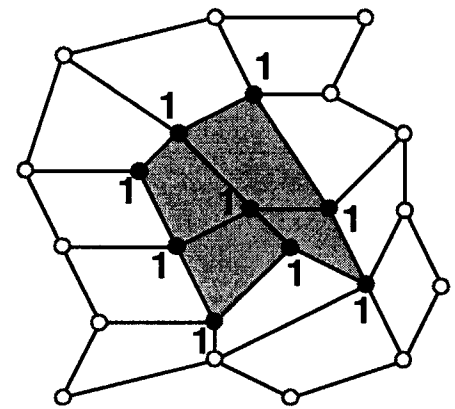
FAST-RKPM Parent Functions

1-D Grid

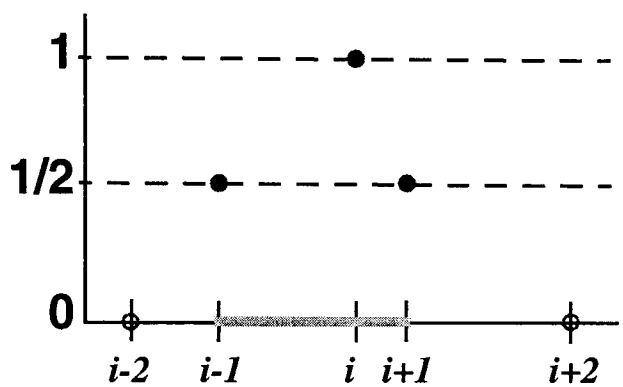
2-D Unstructured Grid

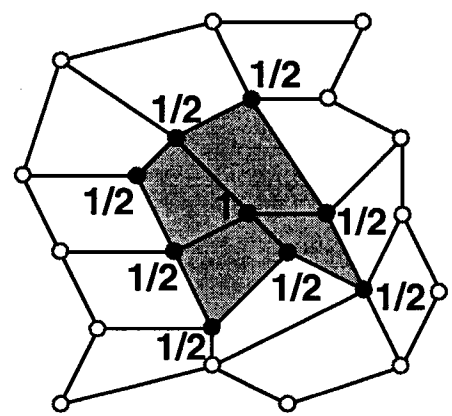
Box





Hat



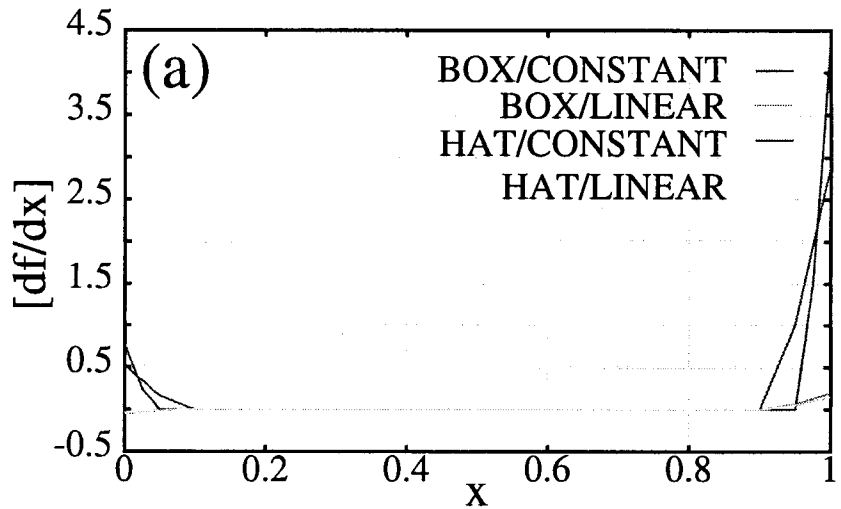


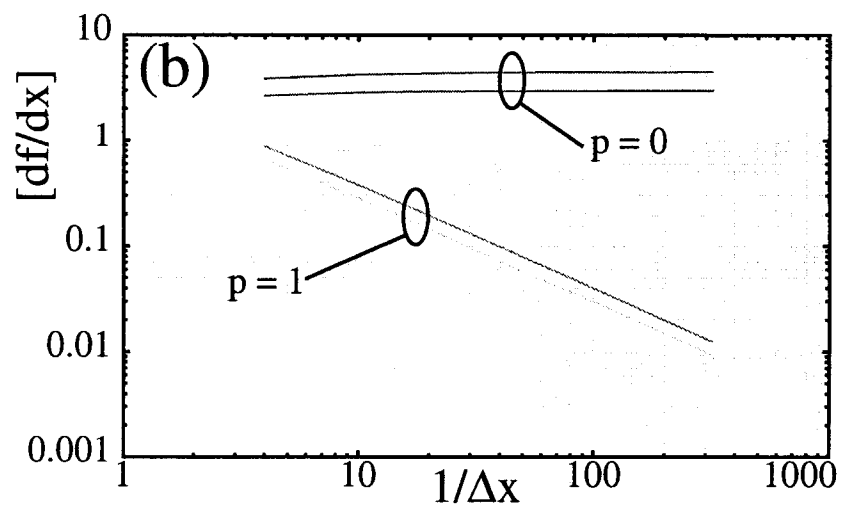
Numerical Experiments Confirm $O(\Delta x^p)$ Convergence of Commutivity Error

Results on uniformly spaced 1-D grid showing commutive error (a) distribution for 21 grid points and (b) convergence rates.

$$f(x) = 1 + x + x^2 + x^3$$

- Uniform moments in domain lead to zero commutivity error. Nonuniform grid will yield non-zero errors.
- Variable moments approaching boundaries yield errors.
- Errors are $O(\Delta x^p)$.





Amplitude Ratio of the Filter is Modified By Boundaries, Consistency and Parent

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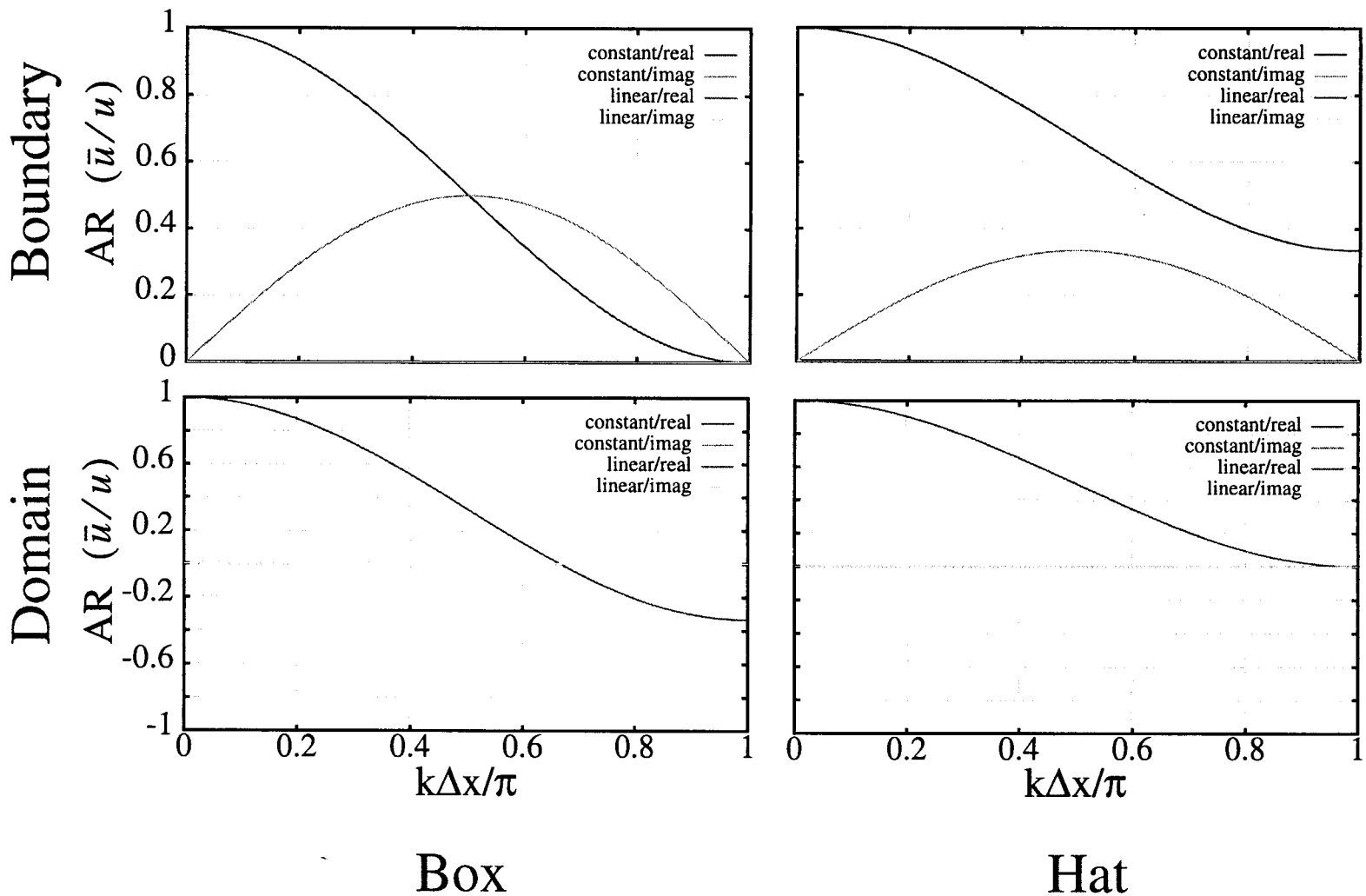

Constant Consistency-Hat Parent Kernel Produces Filter Satisfying Realizability

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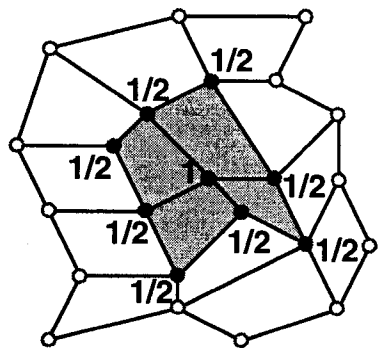
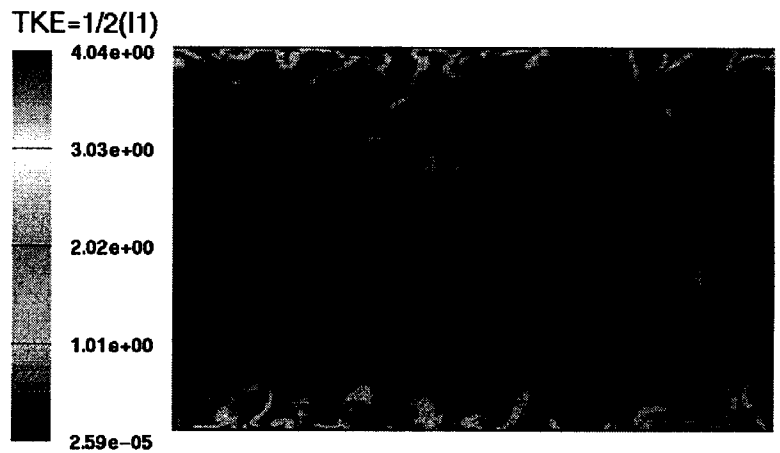
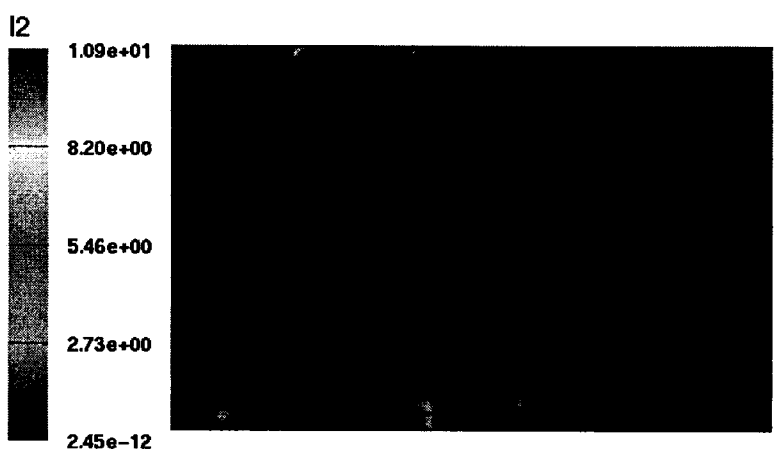
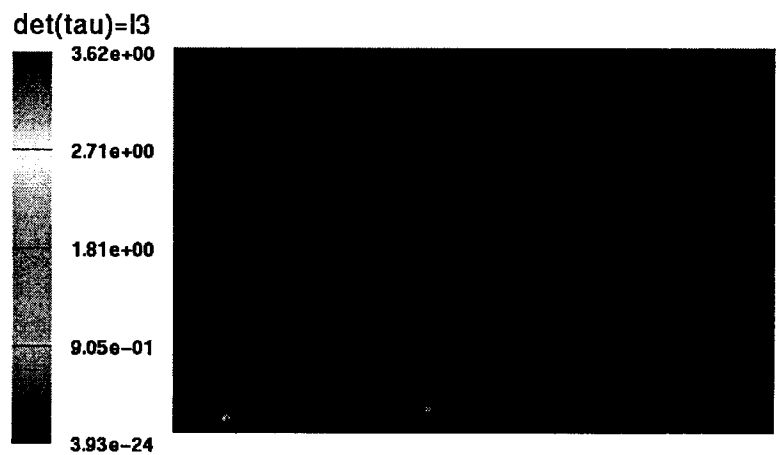
$$P = [1]$$









Linear Consistency-Hat Parent Kernel Produces Filter Satisfying Realizability

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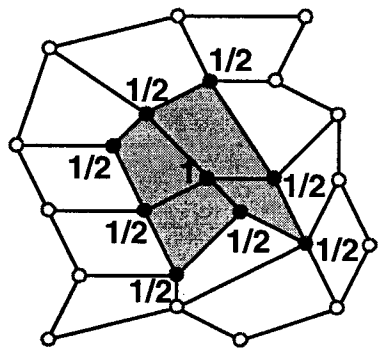
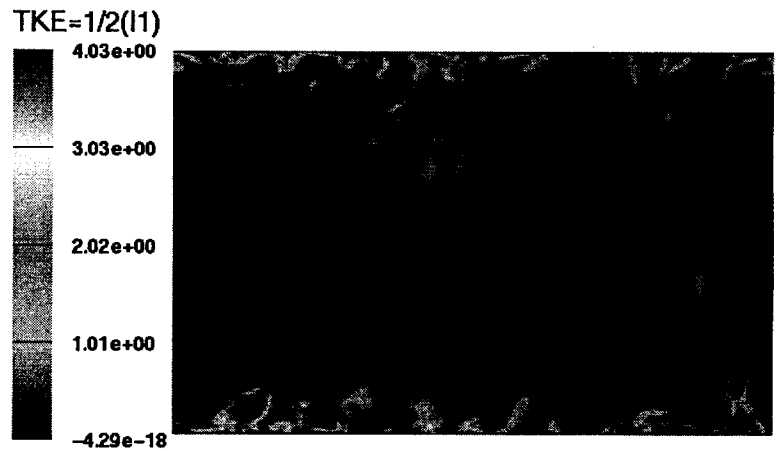
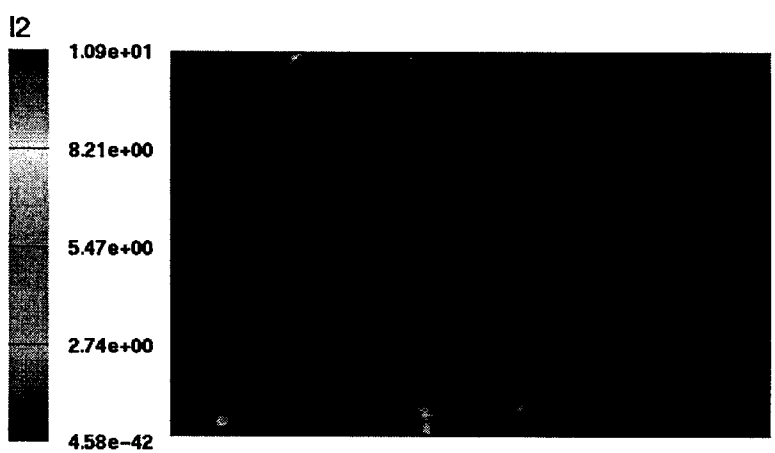
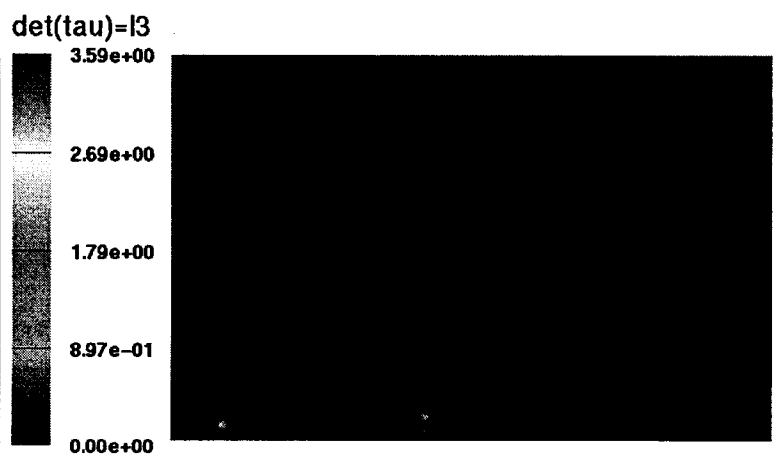
$$P = [\ 1, x, y, z]$$









Constant Consistency-Box Parent Kernel Produces Filter Satisfying Realizability

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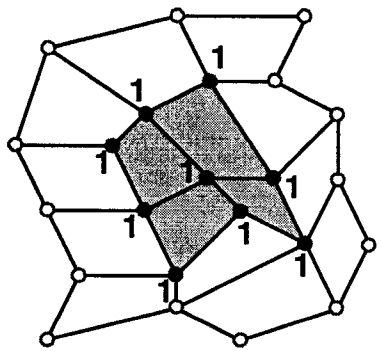
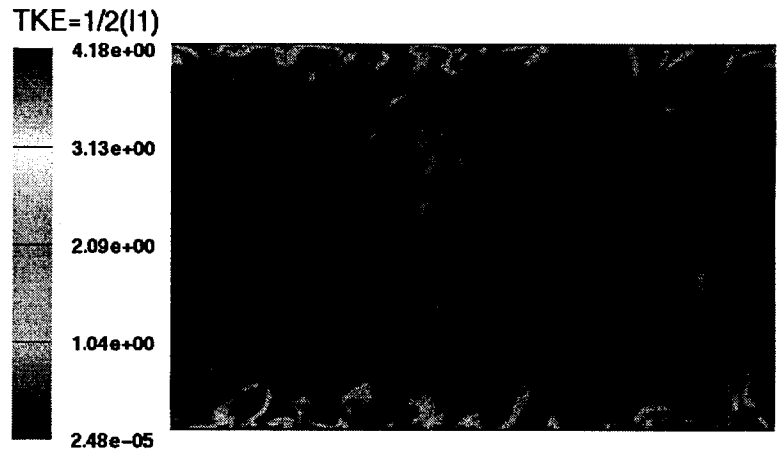
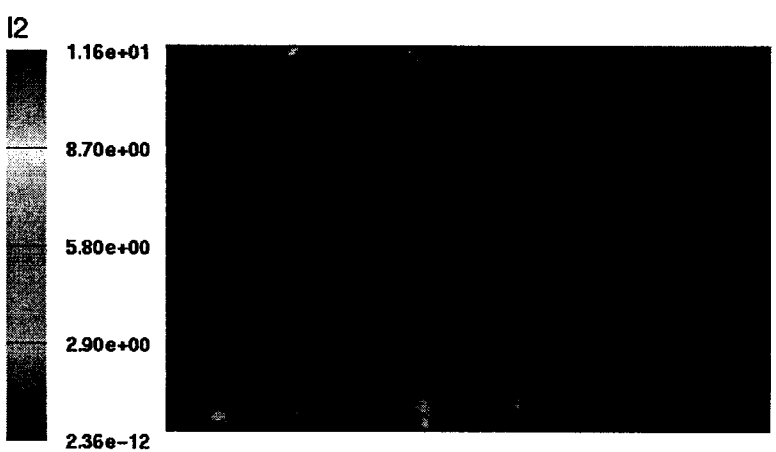
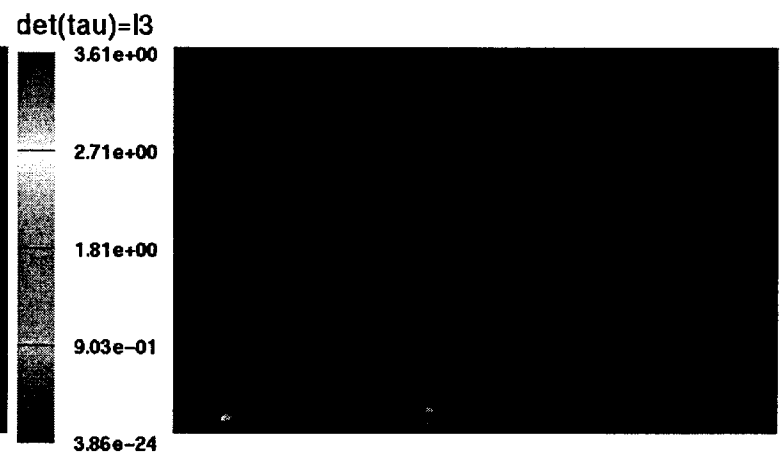
$$P = [1]$$









Linear Consistency-Box Parent Kernel Produces Filter Satisfying Realizability

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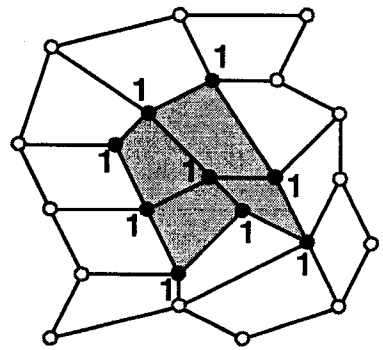
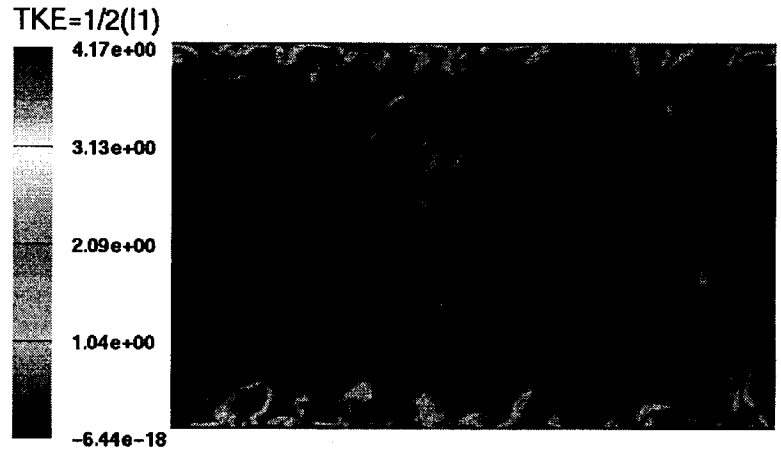
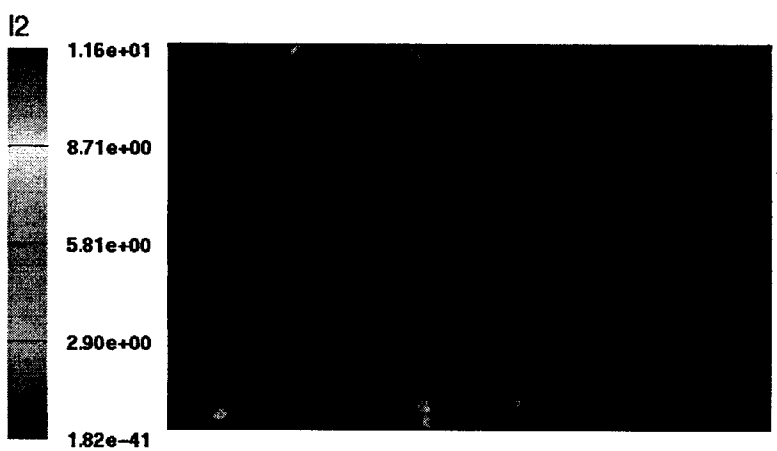
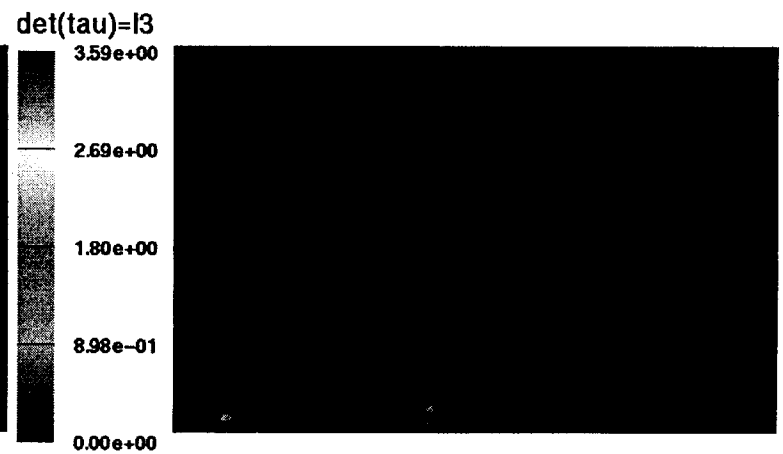
$$P = [\ 1, x, y, z]$$









Change in Consistency Results in Small Change in Invariants

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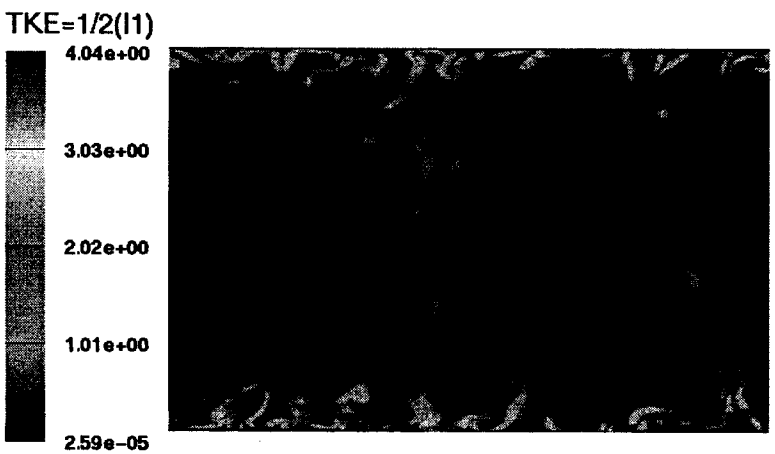
Hat Parent Function

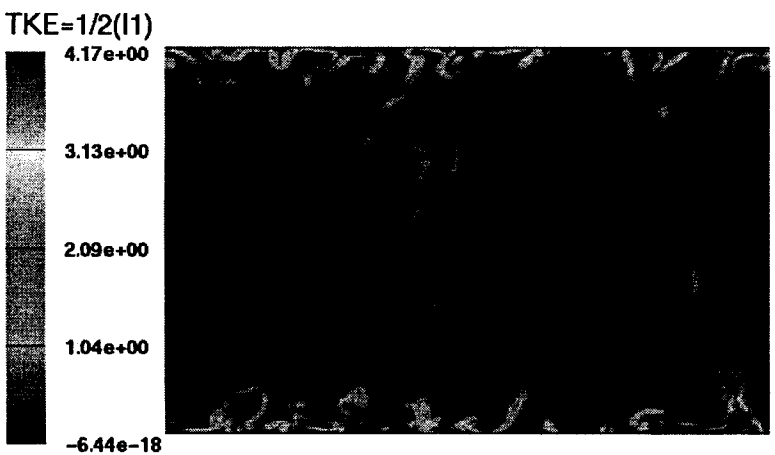


Constant Consistency

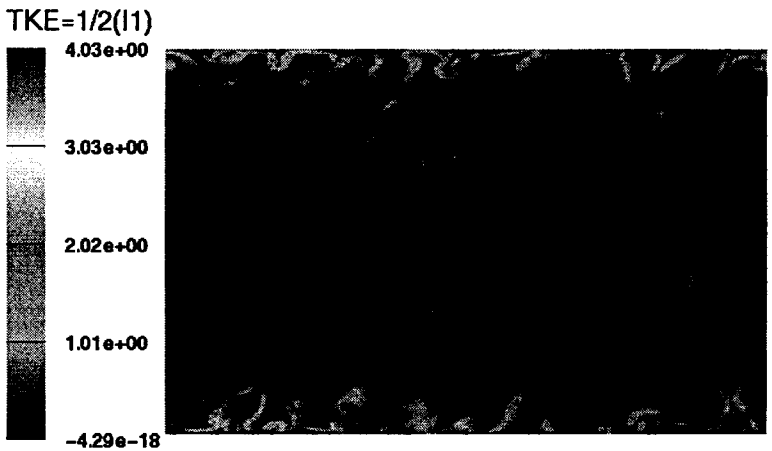
Linear Consistency

Change in Parent Kernel Results in a Significant Change in the Invariants

Linear Consistency



Box Function Parent Kernel



Hat Function Parent Kernel

Conclusions

- A class of Reproducing Kernel Particle Method Filters for filtering of the LES equations have been implemented for unstructured grid FEM calculations.
- Commutivity error of these filters can be shown to depend on the spatial derivatives of the filter moments.
- RKPM controls filter moment derivatives through the reproducing conditions, allowing for an arbitrary level of commutivity accuracy.
- A priori tests demonstrate that for the DNS data used, the RKPM-based filters satisfy the necessary Realizability conditions.