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Results of von Neumann Analyses for Reproducing Kernel Semi-Discretizations*

Thomas E. Voth and Mark A. Christon
Sandia National Laboratories
Albuquerque, New Mexico 87185-0835
tevoth@sandia.gov, machris@sandia.gov

Abstract

The Reproducing Kernel Particle Method (RKPM) has many attractive properties that make it ideal for treating a broad class of physical problems. RKPM may be implemented in a "mesh-full" or a "mesh-free" manner and provides the ability to selectively tune the method, via the selection of dilation parameter and window function, in order to achieve the requisite numerical performance. For example, a reproducing kernel that synchronizes the rates of convergence in the L^2 and H^1 norms can be constructed to yield methods with "built-in" advection stabilization and that avoid locking phenomena in incompressible media. In addition, RKPM provides a framework for performing hierarchical computations making it an ideal candidate for simulating multi-scale problems. Although RKPM has many appealing attributes, the method is quite new and its numerical performance is still being quantified with respect to more traditional discretization methods. In order to assess the performance of RKPM, detailed studies of RKPM on a series of model partial differential equations has been undertaken. The results of von Neumann analyses for RKPM semi-discretizations of one and two-dimensional, first and second order wave equations are presented in the form of phase and group errors. The spatial RKPM discretization employed here uses cubic spline window functions with linear consistency enforced through the use of correction functions [1]. The influence of the dilation parameter and the effects of mass lumping are discussed in the context of phase and group error. The ideal dilation parameter, determined by Liu and Chen's energy approach, is compared to the dilation parameter that minimizes phase-error. In addition, comparative results for the finite difference, finite element and smooth particle hydrodynamics methods are presented. Excellent dispersion characteristics are found for the consistent mass matrix with the proper choice of dilation parameter, i.e., there is essentially no phase or group error for $3\Delta x$ wavelengths. In contrast, the influence of row-sum lumping the mass matrix is shown to introduce severe lagging phase errors. A "higher-order" mass matrix improves the dispersion characteristics relative to the lumped mass matrix but delivers lagging phase errors relative to the fully integrated, consistent mass matrix.

1. W. K. Liu, Y. Chen, S. Jun, J. S. Chen, T. Belytschko, C. Pan, R. A. Uras and C. T. Chang, "Overview and Applications of the Reproducing Kernel Particle Methods," Archives of Computational Methods in Engineering, vol. 3, pp. 3-80, (1996).

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Thomas E. Voth	9113/0835	844-6604	CW Peterson	10/8/97
Mark A. Christon	9231/0819	844-8520		

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