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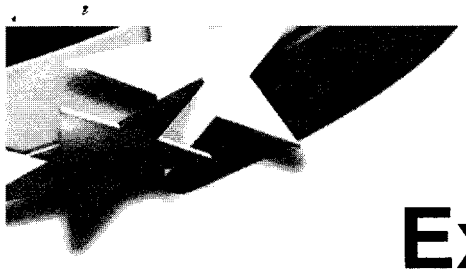

Explicit Transient Dynamics Enforcement of Frictional Sliding Contact in ACME

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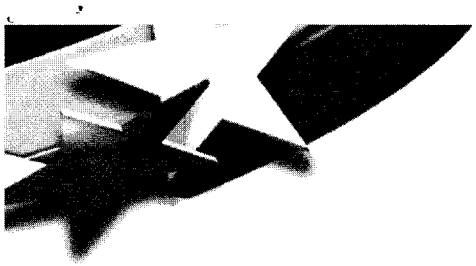

Motivation

- Contact constraints are implicit in nature – introducing coupling to the discretized equations of motion.

- We introduce implementations of frictional sliding contact that iteratively enforce both the normal and frictional contact constraints.

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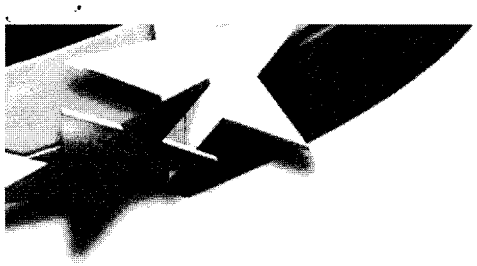

Legacy Algorithm

compute predictor configuration,

$$\mathbf{v}_{n+1/2}^{j=0} = \mathbf{v}_{n-1/2} + \Delta t\, \mathbf{M}^{-1}\left(\mathbf{F}^{e} - \mathbf{F}^{i}(\mathbf{d}_n)\right)$$

$$\mathbf{d}_{n+1} = \mathbf{d}_n + \Delta t\, \mathbf{v}_{n+1/2}^{j=0}$$

compute G_N by contact search

take one pass to impose relative velocity mismatch

$$\lambda_n^{j+1} = \left(\mathbf{H}^{-1}\mathbf{G}_N \mathbf{v}_{n+1/2}^{j}\right)\Big/\Delta t + \lambda_n^{j}$$

$$\mathbf{v}_{n+1/2}^{j+1} = \mathbf{v}_{n+1/2}^{j} - \Delta t \mathbf{M}^{-1}\mathbf{G}_N{}^{T}\left(\lambda_n^{j+1} - \lambda_n^{j}\right)$$

update contact force, $\mathbf{F}_c = \lambda_n^{j}$

$$\mathbf{H} = \mathbf{G}_N \mathbf{M}^{-1}\mathbf{G}_N{}^{T} \approx \mathbf{H}^{\beta} = \begin{bmatrix} \ddots & & \\ & (m_q^s \beta_q)^{-1} & \\ & & \ddots \end{bmatrix}$$

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Algorithm 1: Iterative solution of multipliers λ_n with fixed G_N

compute predictor configuration,

$$\mathbf{v}_{n+1/2}^{j=0} = \mathbf{v}_{n-1/2} + \Delta t\, \mathbf{M}^{-1}\left(\mathbf{F}^e - \mathbf{F}^i(\mathbf{d}_n)\right)$$

$$\mathbf{d}_{n+1} = \mathbf{d}_n + \Delta t\, \mathbf{v}_{n+1/2}^{j=0}$$

compute G_N by contact search

iterate subscript j, beginning with j = 0,

$$\lambda_n^{j+1} = \left(\mathbf{H}^{-1}\mathbf{G}_N \mathbf{v}_{n+1/2}^{j}\right)/\Delta t + \lambda_n^j$$

$$\mathbf{v}_{n+1/2}^{j+1} = \mathbf{v}_{n+1/2}^{j} - \Delta t \mathbf{M}^{-1}\mathbf{G}_N{}^T\left(\lambda_n^{j+1} - \lambda_n^j\right)$$

$$j \leftarrow j+1, \text{ next } j$$

update contact force, $\mathbf{F}_c = \lambda_n^j$

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Algorithm 2: Fully Coupled Constraint Enforcement

iterate subscript k, beginning with k = 0, $F_c^0 = 0$

$$\mathbf{v}_{n+1/2}^{j=0,k} = \mathbf{v}_{n-1/2} + \Delta t \mathbf{M}^{-1}\left(\mathbf{F}^e - \mathbf{F}^i(\mathbf{d}_n) - \left(\mathbf{G}_N^k\right)^T \mathbf{F}_c^k\right)$$

$$\mathbf{d}_{n+1} = \mathbf{d}_n + \Delta t\, \mathbf{v}_{n+1/2}^{j=0,k}$$

compute (by contact search) G_N^k

iterate subscript j, beginning with j = 0,

$$\lambda_n^{j+1} = \left(\mathbf{H}^{-1}\mathbf{G}_N \mathbf{v}_{n+1/2}^{j,k}\right)\Big/\Delta t + \lambda_n^j + \left(\mathbf{G}_N^k\right)^T \mathbf{F}_c^k$$

$$\mathbf{v}_{n+1/2}^{j+1,k} = \mathbf{v}_{n+1/2}^{j,k} - \Delta t \mathbf{M}^{-1}\mathbf{G}_N{}^T\left(\lambda_n^{j+1} - \lambda_n^j\right)$$

$j \leftarrow j+1$, next j

update contact force, $\mathbf{F}_c^{k+1} = \mathbf{F}_c^k + \lambda_n^j$

$k \leftarrow k+1$, next k

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Algorithm 3: Nearly Fully Coupled Constraint Enforcement

compute predictor configuration, $F_c^0 = F_c|_{n-1}$, $G_N^0 = G_N|_{n-1}$

$$\mathbf{v}_{n+1/2}^{j=0} = \mathbf{v}_{n-1/2} + \Delta t \mathbf{M}^{-1}\left(\mathbf{F}^e - \mathbf{F}^i(\mathbf{d}_n) - \left(\mathbf{G}_N^0\right)^T \mathbf{F}_c^0\right)$$

$$\mathbf{d}_{n+1} = \mathbf{d}_n + \Delta t^n \mathbf{v}_{n+1/2}^{j=0}$$

compute G_N by contact search

iterate subscript j, beginning with j = 0,

$$\lambda_n^{j+1} = \left(\mathbf{H}^{-1}\mathbf{G}_N \mathbf{v}_{n+1/2}^j\right) \Big/ \Delta t + \lambda_n^j$$

$$\mathbf{v}_{n+1/2}^{j+1} = \mathbf{v}_{n+1/2}^j - \Delta t \mathbf{M}^{-1}\mathbf{G}_N{}^T\left(\lambda_n^{j+1} - \lambda_n^j\right)$$

$$j \leftarrow j+1, \text{ next } j$$

update contact force, $\mathbf{F}_c = \lambda_n^j$

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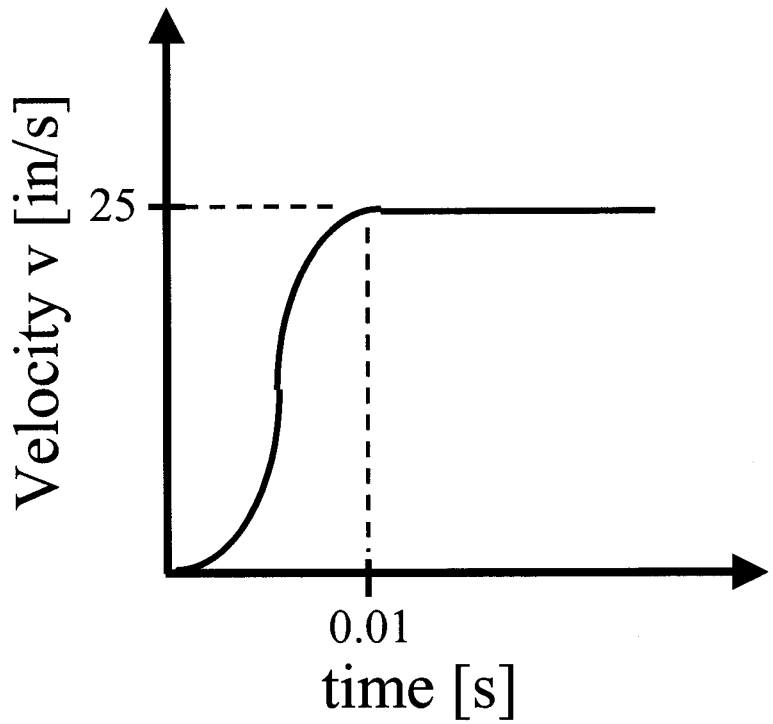
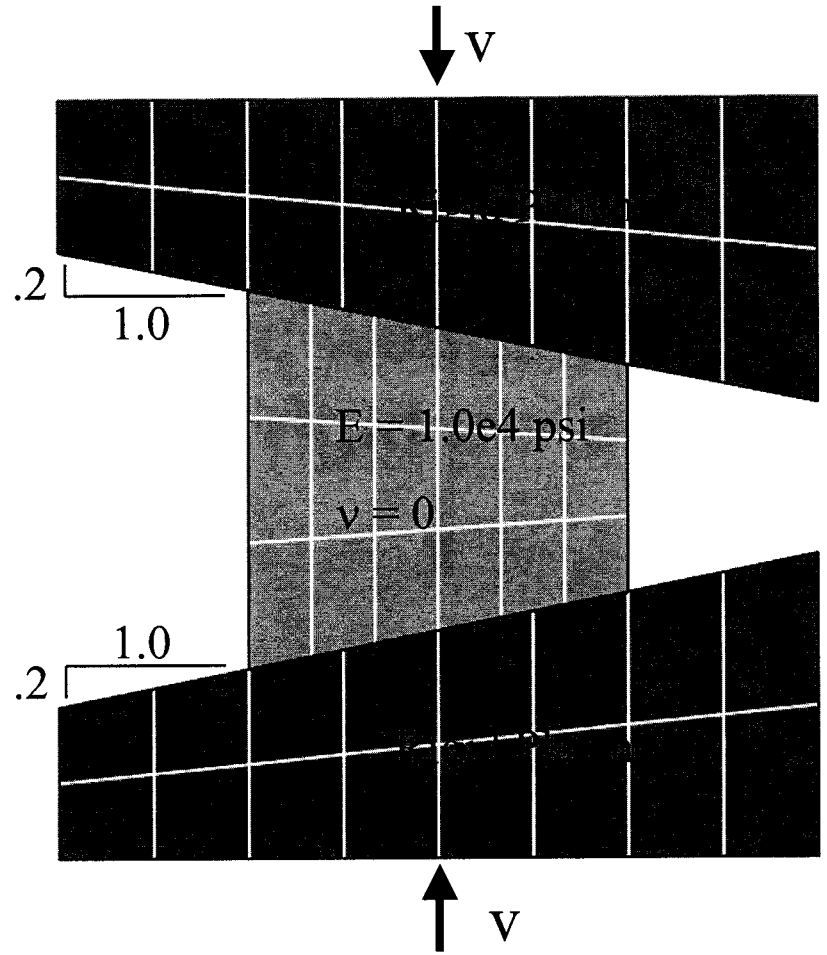
Test Problem: An elastic wedge is compressed between two rigid platens.



Loading history



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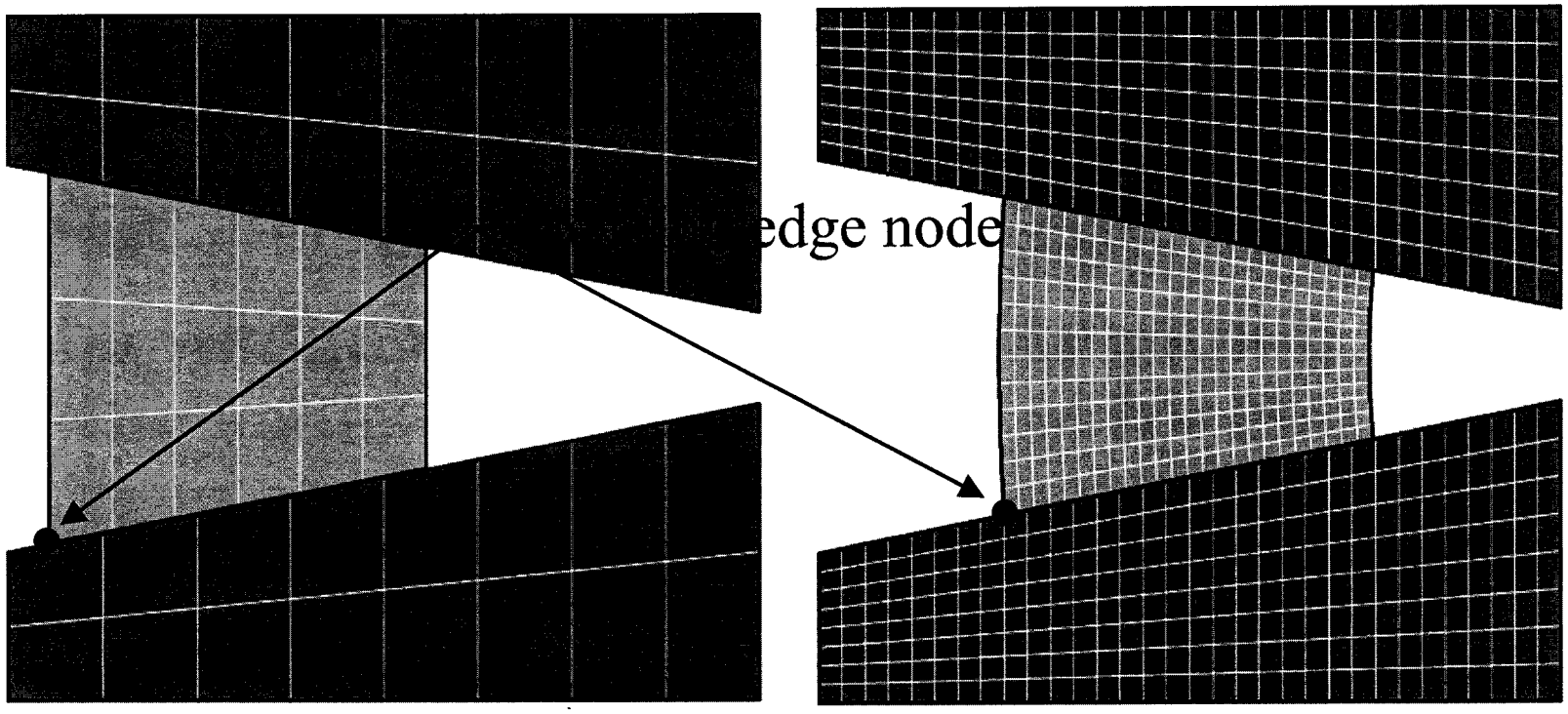
The wedge will slip or stick depending on the value of the coefficient of friction.



$\mu = 0.175 < 0.2$ $\mu = 0.200$

Tests demonstrate wedge slips when $\mu < 0.2$ and sticks for $\mu > 0.2$.

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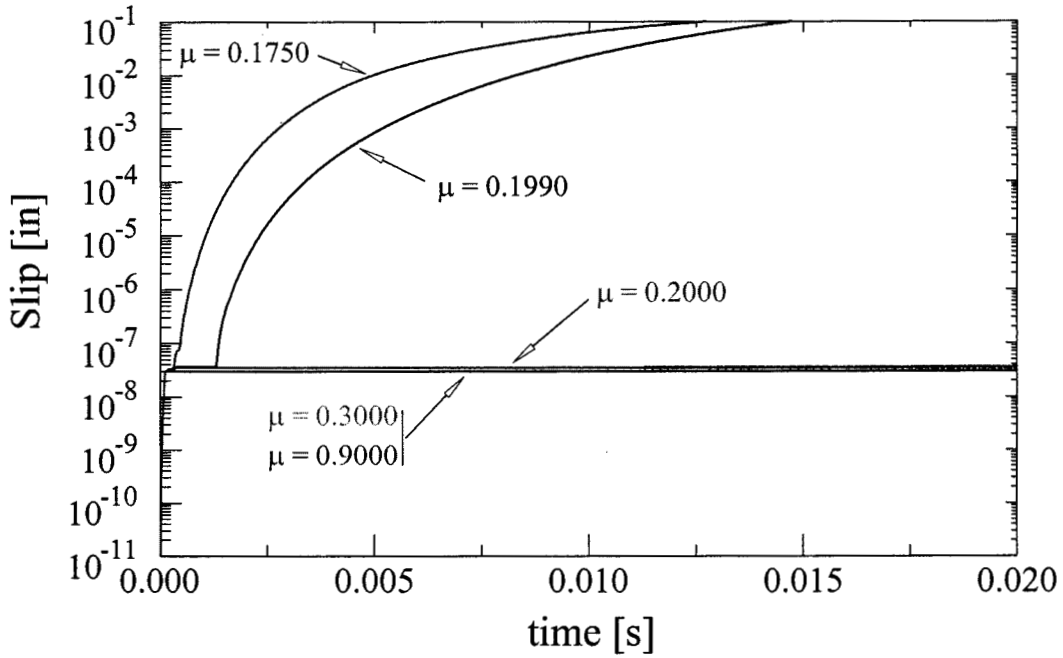
Slip history plot demonstrates slip for $\mu < .2$ and stick for $\mu > 0.2$.



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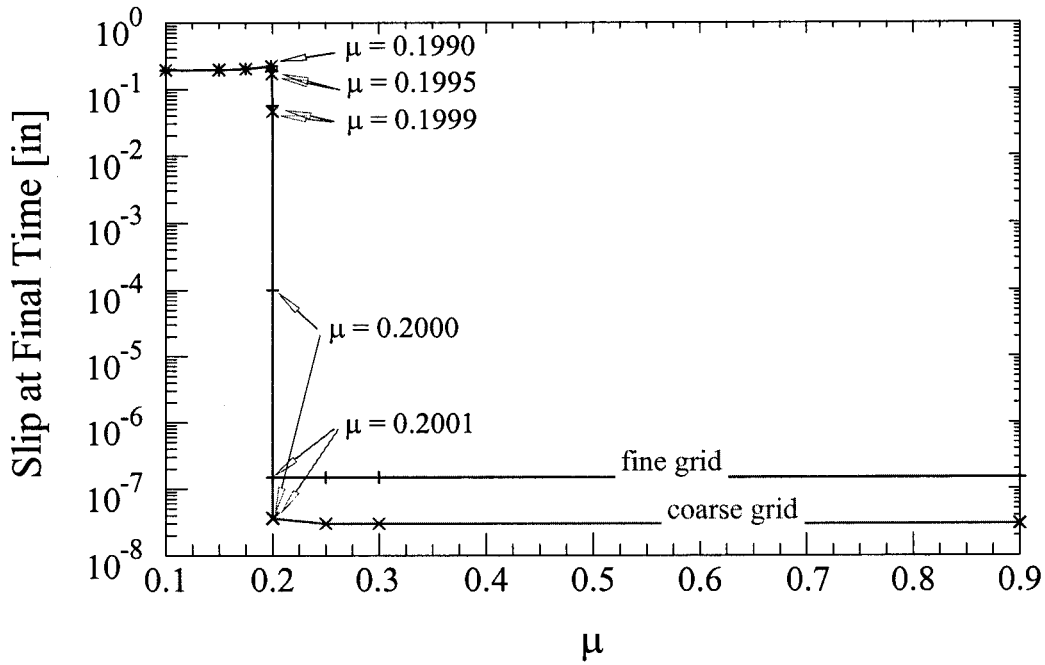
Plot of accumulated slip at time = 0.02 s demonstrates the accuracy of Algorithm 3.



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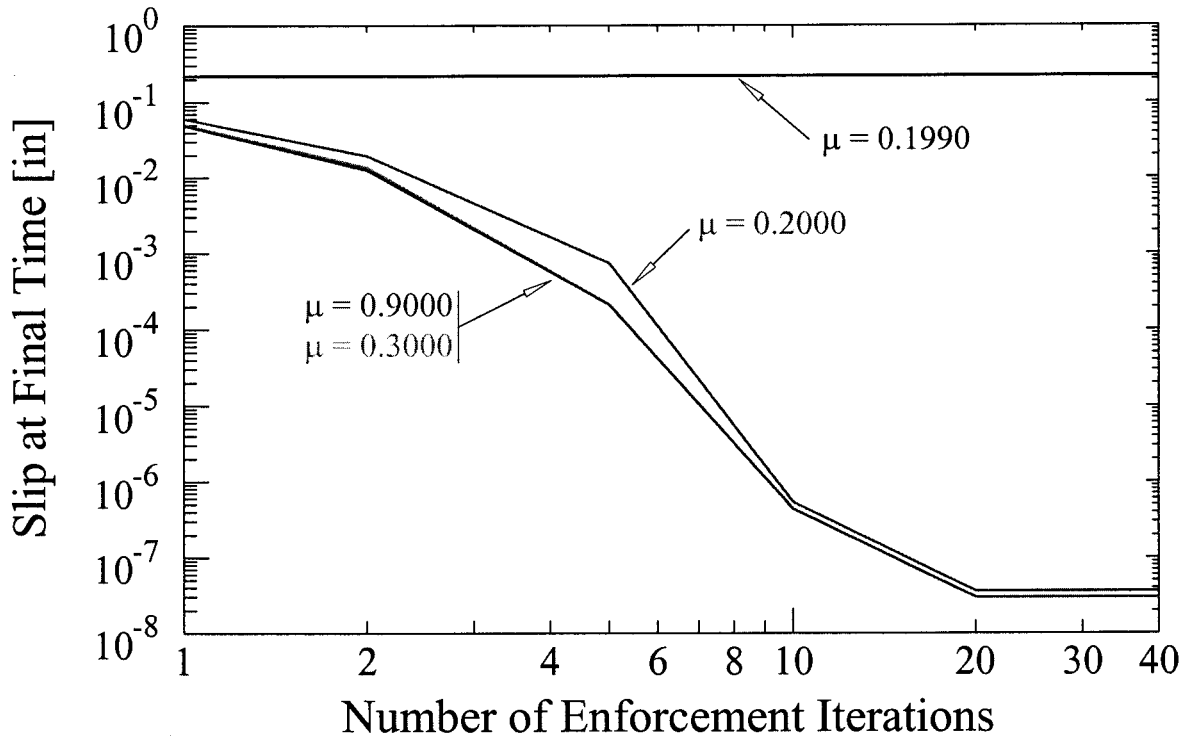
The introduction of iteration clearly increases the accuracy.



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Conclusions

- Four schemes were outlined for enforcement of contact rate constraints:
  - Legacy
  - Iterative Constraint Enforcement with Fixed G_N (Alg. 1)
  - Fully Coupled Constraint Enforcement (Algorithm 2)
  - Nearly Fully Coupled Enforcement (Algorithm 3)
- Algorithm 1 incorporates the dependency of λ on the relative velocity mismatch, but does not include any effect of contact force on the constraint operator (G_N is fixed).
- Algorithm 2 incorporates the deformation dependence of G_N in the solution but requires an expensive search step for each force iteration.
- Algorithm 3 adds to Algorithm 1 a better guess at the initial contact force as well as iteration on contact force.
- The test problem considered here demonstrates the accuracy of Algorithm 3. This accuracy increases with force iterations and is particularly important in the slip regime.

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