SAND2003-0508A

A MULTI-METHODS ASSESSMENT FOR THE ADVECTION-DIFFUSION EQUATION†

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This talk presents a detailed multi-methods comparison of the spatial errors associated with finite difference, finite element and finite volume semi-discretizations of the scalar advection-diffusion equation. The errors are reported in terms of non-dimensional phase and group speeds, discrete diffusivity, artificial diffusivity and grid-induced anisotropy. It is demonstrated that Fourier analysis (aka von Neumann analysis) provides a methodology for separating the spectral behavior of the discrete advective operator into its symmetric dissipative and skew-symmetric advective components. Further it is demonstrated that streamline upwind Petrov-Galerkin and its control-volume finite element analogue, streamline upwind control-volume, introduce both an artificial diffusivity and an artificial phase speed. These terms are in addition to the usual semi-discrete artifacts observed in the discrete phase, group and diffusivity. The generalized Fourier analysis may also be used to extract asympotic estimates of local truncation error without resorting to Taylor series analysis. This permits comparison of methods that are not based on Taylor series, e.g., Galerkin's method, with methods that are. Finally, resolution requirements may be obtained in terms of phase and group speeds, and artificial and discrete diffusivity. Ultimately, this work can be considered a first step in a multi-methods comparison. As such, this work is intended to identify some of the relative strengths and weaknesses of multiple numerical methods in the context of advection-diffusion processes.

† Sandia is a multiprogram laboratory operated by Sandia Corporation, a Lockheed Martin Company, for the United States Department of Energy under Contract DE-AC04-94AL85000